New Worst-Case Response Time Analysis Technique For Real-Time Transactions

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Summary

In this paper, we present a new worst-case response time analysis technique for transactions scheduled by fixed priorities. In the general context of tasks with offsets (general transactions), only exponential methods are known to calculate the exact worst-case response time of a task. The known pseudo-polynomial techniques give an upper bound of the worst-case response time. The new analysis technique presented in this article gives a better (i.e. lower) pseudo-polynomial upper bound of worst-case response time. The main idea of this approach is to combine the principle of exact calculation and the principle of approximation calculation, in order to decrease the pessimism of Worst-case response time analysis, thus allowing to improve the upper bound of the response time provided while preserving a pseudo-polynomial complexity.

1 Introduction

The Response-Time Analysis (RTA) (Audsley et al., 1995) is an essential analysis technique that can be used to perform schedulability tests (i.e. testing if tasks in a system will meet their deadlines). Usually, the task model is an extension of the model of Liu and Layland (Liu et Layland, 1973). The schedulability conditions obtained with the model of (Liu et Layland, 1973) are however too pessimistic for certain kinds of pattern of tasks as tasks with offset (Tindell, 1992, 1994), serial transactions (Traore et al., 2006a), reverse transactions (Traore et al., 2006b), multiframe tasks (Mok et D.Chen, 1996) generalized multiframe tasks (Han et Yan, 1997)(Baruah et al., 1999).

Tindell proposed in (Tindell, 1994) a new model of tasks with offset (transactions) extending the model of Liu and Layland (Liu et Layland, 1973). Transactions are non-concrete(the transaction release times are not fixed a priori), thus the main problems is to determine the worst case configuration for a task under analysis (its critical instant). Offset-Based response time analysis of tasks scheduled under dynamic priorities EDF has been proposed in (Gutierrez et Harbour, 2003). In (Tindell, 1994, 1992) Tindell proposed an exact RTA technique for transactions scheduled by a fixed priorities scheduler, this exact method has an exponential complexity and is intractable for realistic task systems ; In (Tindell, 1994) Tindell has proposed a pseudo-polynomial approximation method providing an upper bound of the worst-case response-time. Later, this approach has been formalized and improved in (Gutierrez et Harbour, 1998) (Maki-Turja et Nolin, 2004a)(Maki-Turja et Nolin, 2004b) (Maki-Turja et Nolin, 2005).

In this paper we combine the principle of exact calculation and the best known approximation calculation, in order to obtain a new analysis technique for tasks with offset scheduled under fixed priorities, which is less pessimistic than the existing techniques. This paper is organized as follow, In Section 2 we present the model of tasks with offsets (a.k.a. transaction), then we review the earlier RTA analysis techniques, the exact analysis method (Tindell, 1992) and the best known approximate analysis method of Nolin (Maki-Turja et Nolin, 2004b, 2005). Then in section 4 we develop the new mixed analysis technique. A performance comparison is presented in section 5.

2 Computational Model

A tasks system Γ is composed of a set of $|\Gamma|$ transactions Γ_i, with $1 \leq i \leq |\Gamma|$ (where $|\Gamma|$ is the number of elements in the set Γ).

$$\begin{array}{rcl} \Gamma & : & \left\{\Gamma_1, \Gamma_2, .., \Gamma_{|\Gamma|}\right\} \\ \Gamma_i & : & \left\{\tau_{i1}, \tau_{i2}, ..., \tau_{i|\Gamma_i|}, T_i\right\} \\ \tau_{ij} & : & < C_{ij}, O_{ij}, D_{ij}, J_{ij}, B_{ij}, P_{ij} > \end{array}$$

Each transaction Γ_i (see figure 1) consists of a set of $|\Gamma_i|$ tasks τ_{ij} released at the same period T_i , with $0 < j \leq |\Gamma_i|$. Without loss of generality, we suppose that the tasks are ordered in the set by increasing offset. A task τ_{ij} is defined by : a worst-case execution time (WCET) C_{ij}, an offset O_{ij} related to the release date of the transaction Γ_i, a relative deadline D_{ij}, a maximum jitter J_{ij} (the activation time of task τ_{ij} may occur at any time between $t_0 + O_{ij}$ and $t_0 + O_{ij} + J_{ij}$, where t_0 is the release date of the transaction Γ_i), a maximum blocking factor B_{ij} due to lower priority tasks (e.g. priority ceiling protocol (Sha et al., 1990)), and P_{ij} is its priority (we assume a fixed-priority scheduling policy). The figure 1 presents an example of transaction Γ_i composed of three tasks with period $T_i = 16$. Note that each transaction is non-concrete (in fact it's sporadically periodique). Let us note $hp_i(\tau_{ua})$ the set of indices of the tasks of Γ_i with a priority higher than the priority of a task under analysis τ_{ua}, assuming that the priorities of the tasks are unique.

3 Response Time Analysis

In this section, we present the related work on RTA for tasks with offsets scheduled under fixed priorities. A critical instant corresponds, for a task under analysis τ_{ua}, to the worst case scenario for this task. In the case of classic tasks, the critical instant correspond to the simultaneous activation of all the higher priority tasks with τ_{ua}, Then we consider, starting from this worst-case scenario, a time interval when the processor never goes idle. This interval is

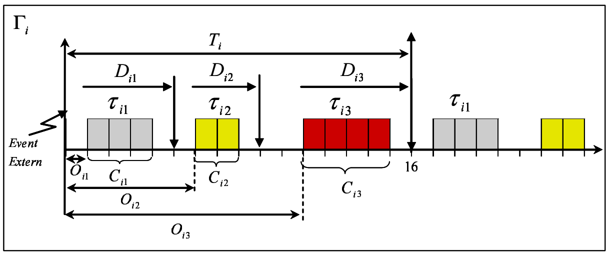

FIG. 1 – *Example of transaction.*

called a busy period, and gives the WCRT of the task under analysis. For transactions, Tindell showed that the critical instant of a task under analysis, noted τ_{ua}, is a particular instant when it is released at the same time as one task of higher priority in each transaction.

In order to simplify and clarify the computation formulas for the analysis methods, we will consider in the sequel of this paper that the task under analysis τ_{ua} is the only task of the transaction Γ_u and that it has only one instance activated in any busy period of length t. This assumption can be removed later by using classic RTA method.

Thus a critical instant coincides with the simultaneous activations of a candidate task τ_{ic_i} (task of higher priority than task under analysis) of each transaction Γ_i. The response time R_{ua} of the task under analysis τ_{ua} can be calculated by iterative fix-point lookup. We note $W_{ic_i}(t)$ is the interference of a transaction Γ_i in the busy period of length t, when a candidate task τ_{ic_i} initiate the critical instant. Φ_{ijc} is the phasing between a task τ_{ic} , and a critical instant candidate initiated by the candidate task τ_{ic} ; i.e the first instance of a task τ_{ic} (activated after the critical instant) will be released at Φ_{ijc} time units after the critical instant, and subsequent releases will occur periodically every T_i.

$$R_{ua} = C_{ua}$$

$$R_{ua} = C_{ua} + \sum_{\forall i \neq u} W_{ic_i}(\tau_{ua}, R_{ua}) \tag{1}$$

$$Where : W_{ic}(\tau_{ua}, t) = \sum_{\forall j \in hp_i(\tau_{ua})} \left(\left\lfloor \frac{J_{ij} + \Phi_{ijc}}{T_i} \right\rfloor + \left\lceil \frac{t - \Phi_{ijc}}{T_i} \right\rceil \right) C_{ij} \tag{2}$$

$$\Phi_{ijc} = (O_{ij} - (O_{ic} + J_{ic}) \; mod \; T_i \tag{3}$$

The main problem of RTA technique of tasks with offsets is that we don't know which task τ_{ic_i} of each transaction Γ_i must be considered to create the worst-case busy period. In fact, the choice of this task candidate in each transaction depends on the length of the busy period. An exact calculation method (Tindell, 1994) would require to evaluate the response time obtained by carrying out all the possible combinations of the tasks of priority higher in each transaction and to choose the task in each transaction that leads to the worst-case response time.

$$R_{ua} = \max_{\forall i \neq u \; and \; \forall c_i \in hp_i(\tau_{ua})} R_{ua} \tag{4}$$

This exhaustive method has an exponential complexity and is intractable for realistic task systems. In order to avoid this problem, several approximation methods giving an upper bound of the worst-case response time have been proposed. The best known approximation method is the one based on the "imposed interference" (Maki-Turja et Nolin, 2004b, 2005).

3.1 Upper-Bound Approximation For WCRT

Tindell proposed in (Tindell, 1994) an approximate analysis technique used to obtain an upper bounds for the worst-case response times in a system of transactions scheduled under fixed priorities. This technique calculates an upper bound of the interference of the tasks of a transaction Γ_i in a busy period of duration t, as the maximum of all possible interferences that could have been caused by considering each of the tasks of Γ_i as the one originating the busy period.

$$\begin{aligned} R_{ua} &= C_{ua} \\ R_{ua} &= C_{ua} + \textstyle\sum_{i\neq u}(W_i(\tau_{ua}, R_{ua})) \qquad (5) \\ Where: W_i(\tau_u, t) &= \max_{\forall c\in hp_i(\tau_{ua})} W_{ic}(\tau_u, t) \qquad (6) \end{aligned}$$

This method is not exact, but has a pseudo-polynomial complexity, which makes it applicable even for relatively large systems. A sufficient test of schedulability is given by this method, if the response times obtained are smaller than the respective deadlines, the system is schedulable, if not, no definitive answer can be given.

Nolin (Maki-Turja et Nolin, 2004b) improved the approximative method by introducing the imposed interference concept. This method consists in calculating the interference effectively imposed by a task τ_{ij} on a lower priority task τ_{ua} during a time interval of length t; the underlying idea is that the interference of a higher priority task can't exceed t in a time interval of length t. In order to calculate the "‘imposed interference"’ (Maki-Turja et Nolin, 2004b) remove the unnecessary overestimation (parameter x_{ijc} in the formula) taken into account in the classic computation of the interference imposed by a task τ_{ij} on a lower priority task τ_{ua}. This overestimation does not have any impact in the case of tasks without offset but has a considerable effect in the approximation of the worst-case response time when we are in the presence of tasks with offsets.

Let us note $W_{ijc}(t)$ (resp,$W_{ic}(t)$) the interference that τ_{ij} (resp,Γ_i) imposes effectively on the response time of τ_{ua} during a time interval of length t when τ_{ic} is released at the critical instant.

$$W_{ic}(\tau_{ua}, t) = \sum_{\forall j\in hp_i(\tau_{ua})} W_{ijc}(t) \qquad (7)$$

Where :

$$\begin{aligned} W_{ijc}(t) &= \left(\left\lfloor \frac{J_{ij}+\Phi_{ijc}}{T_i} \right\rfloor + \left\lceil \frac{t^*}{T_i} \right\rceil\right) C_{ij} - x_{ijc} \\ t^* &= t - \Phi_{ijc} \\ \Phi_{ijc} &= (T_i + (O_{ij} - O_{ic}))\, Mod\, T_i \\ x &= \begin{cases} C_{ij} - (t^*\ mod\ T_i) & \text{if } t^* > 0 \wedge (0 < t^* mod T_i) < C_{ij}) \\ 0 & \text{otherwise} \end{cases} \end{aligned}$$

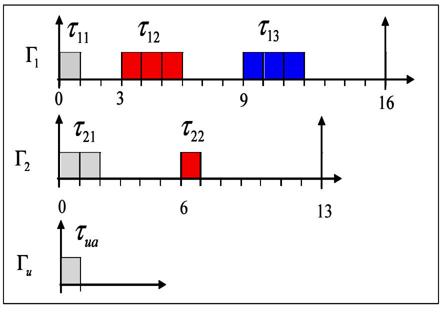

FIG. 2 – *Example of a system of transactions*

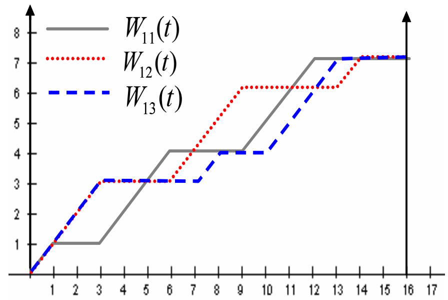

FIG. 3 – *Imposed interferences of* Γ_1

x_{ijc} corresponds to the part of the task τ_{ij} that cannot be executed in the time interval of length t ; since this interference is not effectively imposed in this interval, it is not taken into account (note that this part is the main difference between the methods presented in (Gutierrez et Harbour, 1998) and (Maki-Turja et Nolin, 2004b). The evolution of the imposed interference function in the time can be presented by a curve with slanted stairs, as it is showed in the figure 3 for the interference function of transaction Γ_1 of the system (figure 2).

An efficient implementation of this approximation method has been proposed in (Maki-Turja et Nolin, 2005) ; it is using a static representation of the periodic interference function, and during the response-time calculation, it is uses a simple lookup function in order to compute its value. We apply this technique to obtain an upper bound for the response time of a task τ_{ua} presented on the figure 2.

3.1.1 Example

Note in $R_{ua}^{(i)}$, (i) denotes the step in the fix-point lookup of $R_{ua}^{(n)} = R_{ua}^{(n+1)}$

$$
\begin{aligned}
\mathrm{R}_{ua}^{(0)} &= C_{ua} = 1 \\
\mathrm{R}_{ua}^{(1)} &= C_{ua} + W_1(1) + W_2(1) = 1 + 3 + 2 = 6 \\
\mathrm{R}_{ua}^{(2)} &= C_{ua} + W_1(6) + W_2(6) = 1 + 4 + 2 = 7 \\
\mathrm{R}_{ua}^{(3)} &= C_{ua} + W_1(7) + W_2(7) = 1 + 4 + 3 = 8 \\
\mathrm{R}_{ua}^{(4)} &= C_{ua} + W_1(8) + W_2(8) = 1 + 6 + 3 = 10 \\
\mathrm{R}_{ua}^{(5)} &= C_{ua} + W_1(10) + W_2(10) = 1 + 6 + 3 = 10
\end{aligned}
$$

In order to introduce less pessimism in the value of the upper bound obtained by this method, we try to locate the source of the pessimism in the case of the earlier example, this diagnostic will be the base of our method developed in section 4.

3.2 Pessimism of approximative approach

In the approximative analysis, the pessimism on response times is produced by the interference function of approximation $W_i(\tau_u, t)$ which takes the interference of the tasks of a transaction Γ_i in a busy period of duration t, as the maximum of all possible interferences that could have been caused by considering each of the tasks of Γ_i as the one initiating the critical instant. The drawback of this function is the change of the task initiating the critical instant during the iterative calculation of the WCRT. For a given analyzed task τ_{ua}, the pessimism can be produced by the application of the approximative interference function $W_i(\tau_{ua}, t)$ on one or more transactions.

In the previous example, the exact value of the WCRT of τ_{ua} equals 8. At this instant, there is a change of the candidate task of transaction Γ_1, before the instant 8 the maximum interference of Γ_1 corresponds to its interference when a task τ_{11} initiates the critical instant, but after this date it is the task τ_{12} which initiates the critical instant. Thus the pessimism is produced when at the instant corresponding to the exact WCRT there is a change of a candidate task which initiates the critical instant for any transaction of the system.

Therefore, using an exact interference function on transaction Γ_1 and an approximative interference function on the transaction Γ_1 in the calculation of the worst-case response time, we can reduce the pessimism of the upper bound obtained by the approximative approach.

candidate task τ_{11} *:*

$$\begin{aligned}
\mathrm{R}^{(0)}_{ua,11} &= C_{ua} = 1 \\
\mathrm{R}^{(1)}_{ua,11} &= C_{ua} + W_{11}(1) + W_2(1) = 1 + 1 + 2 = 4 \\
\mathrm{R}^{(2)}_{ua,11} &= C_{ua} + W_{11}(4) + W_2(4) = 1 + 4 + 2 = 7 \\
\mathrm{R}^{(3)}_{ua,11} &= C_{ua} + W_{11}(7) + W_2(7) = 1 + 4 + 3 = 8 \\
\mathrm{R}^{(4)}_{ua,11} &= C_{ua} + W_{11}(8) + W_2(8) = 1 + 4 + 3 = 8
\end{aligned}$$

For candidate task τ_{12} *:* $R_{ua,12} = 6$ and *for candidate task* τ_{13} *:* $R_{ua,13} = 6$

The maximum of the values obtained is $R_{ua} = 8$ that represent an upper bound for the WCRT of τ_{ua}. This upper bound for WCRT is less pessimistic than the one obtained by the approximative analysis ($R_{ua} = 10$). Thus the number of cases that need to be checked corresponds to the number of candidate tasks of Γ_1. We use this idea as a basis of our mixed response times analysis technique that is developed in the next section.

4 Mixed Response Time Analysis Technique

This new technique will let us obtain an upper bound for the worst-case response times for transactions systems with fixed priorities. It has a pseudo-polynomial complexity which makes it applicable for relatively large systems, and it's tunable. The more steps allowed, the better the quality of the test is.

$$\begin{aligned}
&R_{ua,ic} = C_{ua} \\
&R_{ua,ic} = C_{ua} + W_{ic}(R_{ua}) + \sum_{\Gamma_k \in \Gamma, \Gamma_k \neq \Gamma_i} (W_k(R_{ua})) \\
&W_k(t) = \max_{c \in hp_k(\tau_{ua})} W_{kc}(t)
\end{aligned} \tag{8}$$

For a given transaction Γ_i, we apply an exact function of interference, while an approximation function of interference is applied for all others transactions of the system. In this case we have to evaluate the response time obtained by applying an exact interference Γ_i when each candidate task τ_{ic} initiates the critical instant, to choose the maximum as the upper bound for the response time of the task under analysis τ_{ua}. Let us note $R_{ua,ic}$ the response time of τ_{ua} obtained when we take into account the exact interference of Γ_i when the candidate task τ_{ic} initiates the critical instant.

The interference of the transaction Γ_k ($W_{kc}(t)$) when τ_{kc} initiate the critical instant is calculated by using the imposed interference function presented in the precedent section. $R_{ua,i}$ the upper bound for WCRT of τ_{ua} is obtained as the maximum of response times $R_{ua,ic}$.

$$R_{ua,i} = \max_{c \in hp_i(\tau_{ua})} R_{ua,ic} \tag{9}$$

Theorem 1 *(For proof see (Rahni et al., 2007))*
the value of response time $R_{ua,i}$ obtained by this mixed method (corresponding to a transaction Γ_i for which an exact interference function is applied) is always between the exact value of WCRT and the upper bound calculated by the approximate method of Tindell-Nolin.

This method is applied for one transaction Γ_i of a system, and the upper bounds for WCRT provided is sure (is never lower than the exact value of WCRT). In order to obtain the best upper bound (the smallest upper bound), we need to calculate the upper bounds ($R_{ua,i}$) corresponding to all the transactions of the system, thus to choose the minimum (best) as an upper bound for the response time of the task under analysis τ_{ua}. In this case the number of scenarrii that need to be checked equals the number of transactions in the system.

$$R_{ua} = \min_{i \in 1..|\Gamma|, i \neq u} R_{ua,i} \tag{10}$$

Note that this method provide an upper bound for WCRT which less pessimistic than provided by the approximated method (presented in section 2). The total number of cases to check corresponds to the number of higher priority tasks in the system.

4.1 Example

We apply the mixed method on an example presented in figure 4. The exact value of WCRT of a task under analysis τ_{ua} equals 20 times unites. The upper bound for WCRT of τ_{ua} is 28 units of times. The table of figure 5 resume the different steps of calculation of the WCRT by the mixed method. The upper bound for WCRT of τ_{ua} obtained by the mixed method is 20 unites of times, it equals the exact value of WCRT. In this case note that with Nolin's method the obtained value is 28.

For candidate task τ_{11} :

$$\begin{aligned}
\mathrm{R}_{ua,11}^{(0)} &= C_{ua} = 1 \\
\mathrm{R}_{ua,11}^{(1)} &= C_{ua} + W_{11}(1) + W_2(1) + W_3(1) &&= 1+3+2+2 = 8 \\
\mathrm{R}_{ua,11}^{(2)} &= C_{ua} + W_{11}(8) + W_2(8) + W_3(8) &&= 1+12+4+3 = 20 \\
\mathrm{R}_{ua,11}^{(3)} &= C_{ua} + W_{11}(20) + W_2(20) + W_3(20) &&= 1+12+4+3 = 20
\end{aligned}$$

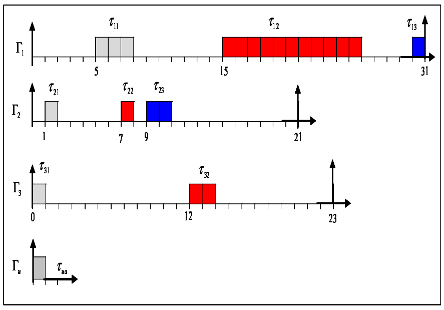

FIG. 4 – *System of transactions.*

Γ_i	Candidate task τ_{ic}	$R_{ua,ic}$	$R_{ua,i}$	R_{ua}
Γ_1	τ_{11}	20	20	20
	τ_{12}	11		
	τ_{13}	9		
Γ_2	τ_{21}	19	23	
	τ_{22}	23		
	τ_{23}	23		
Γ_3	τ_{31}	28	28	
	τ_{32}	27		

FIG. 5 – *Response time with mixed analysis*

Note that in the case of systems composed of two transactions the Values of WCRT provided are exact, because in this situation the mixed method is equivalent to the exact method.

4.2 Tunable Mixed Method

Because the pessimism on the response times is produced by the application of approximative interference function on one or several transactions in the system. Using the principle of the mixed analysis technique, in order to reduce the pessimism of an upper bound for WCRT, we will vary the number of transactions E for which we apply an exact interference function. For example the number of transactions for which an exact calculation is applied equals 1 ($E = 1$) for the precedent method. For a number of exact transactions equal to E, the response times obtained for the systems composed of a number of transactions lower than $E + 1$ is exact ; i.e with no pessimism.

Since the mixed method of WCRT analysis is based on the efficient implementation algorithm presented in (Maki-Turja et Nolin, 2005), then the time complexity of the mixed method is the same as Nolin's with a difference in the number of the critical instants considered. Note that the complexity of the original method (Gutierrez et Harbour, 1998; Maki-Turja et Nolin, 2004b) is $O(x\ |\Gamma_i|^3)$, where x is the number of steps used in the fix-point calculation. (Maki-Turja et Nolin, 2005) showed that the complexity of their method was $O(|\Gamma_i|^3 + x\ log\,|\Gamma_i|^2)$. In the mixed method we test K critical instants candidate, the complexity is $O(|\Gamma_i|^3 + K\ x\ log\,|\Gamma_i|^2)$. We note E the number of transactions for which we apply an exact interference function.

$$K = \frac{|\Gamma_i|!}{E!\ (|\Gamma_i| - E)!}\ |\Gamma_i|^E$$

$\frac{|\Gamma_i|!}{E!\ (|\Gamma_i| - E)!}$ corresponds to the choice of E transactions for which we want an exact RTA, and $|\Gamma_i|^E$ corresponds to the number of scenarii to explore for each choice of E transactions.

For example, for $E = 1$ the complexity of the method is $O(|\Gamma_i|^3 + x\ |\Gamma_i|^2\ log\,|\Gamma_i|^2)$, and for $E = 2$ it is $O(|\Gamma_i|^3 + x\ |\Gamma_i|^4\ log\,|\Gamma_i|^2)$

Thus the pessimism of the mixed method decreases while increasing the number of transaction for which we apply an exact interference function. In the simulation we have implemented the mixed method for a number of exact transaction varying for $E = 1$ to $E = 3$. Since the complexity of the mixed method increases with the number of transactions (E) on which an exact calculation is applied. and according to the simulation results (pessimism and treatment times), we can adopt the mixed method with $E = 1$ or $E = 2$.

5 Performance Evaluation

In order to evaluate and quantify the improvement made on worst-case response time by of our mixed method compared to existing methods We have implemented the following algorithms :

- NM1 : the mixed method with the number of transaction for which we apply an exact interference function equal to $E = 1$
- NM2 : the mixed method with $E = 2$
- NM3 : the mixed method with $E = 3$
- Nolin : the approximative method of Nolin (Maki-Turja et Nolin, 2005)
- Exact : the exact analysis (Tindell, 1994; Gutierrez et Harbour, 1998)

In the implementation all the methods, we have used an efficient implementation proposed by Nolin (Maki-Turja et Nolin, 2005). The tests carried out correspond to the calculation of the response time of all the tasks of the transactions by using the complete set of response-times (for all instances of τ_{ua} released in the busy period). Each point in each graph has been obtained by taking the mean value of 100 randomly generated transactions systems.

- Random generator characteristics : The random generator of transactions systems takes the following parameters as input : Total system load, Number of transaction per system and Number of tasks per transaction. Using these parameters the others properties of task systems are generated :

- Using the UUniFast algorithm presented in (Bini et Buttazzo, 2004), the total system load is proportionally distributed over all transactions.
- Periods of transactions (Ti) are randomly distributed in the range 100 to 1.000.000 time units (uniform distribution).
- Each offset (O_{ij}) is randomly distributed within the transaction period (uniform distribution).
- Using the UUniFast algorithm presented in (Bini et Buttazzo, 2004), the transaction load is proportionally distributed over all tasks. The execution times (C_{ij}) are calculated using the periods of transaction and a task load.
- The blocking factors B_{ij} and Jitters J_{ij} are nulls.
- The priorities are assigned in deadline monotonic order.

- Criteria of comparison :

- Pessimism : The pessimism of a method M is $\left(R_{ua}^{M} - R_{ua}^{Exact}\right) / R_{ua}^{Exact}$, which is giving how pessimistic the obtained WCRT is pessimistic. Of course the lower the better.
- Execution time : the time required by the method M for computing the WCRT.

Figures 6,7,8,9 correspond to a base configuration where the system load is 80%, and there are 6 tasks per transaction. From this basic configuration we change the number of transactions. These figures show that the difference between the fraction of pessimism of the mixed methods and approximative method increases as the number of tasks per transaction grows. But the pessimism increases slower for the mixed methods than for Nolin's method. For example we can see in these graphs that for more than 10 transactions of 5 tasks, the maximum pessimism for Nolin's method is around 8%, while for mixed methods (NM1 and NM2) the pessimism does not exceed 2%.

In the figures 8,12 we compare the fraction of tasks concerned by the pessimism. This fraction measures the number of tasks in a transactions system for which a response times provided by the analysis techniques is pessimistic (greater than the exact value of WCRT). The fraction of tasks with pessimism increases with the number of transactions in the system, and with the number of tasks per transaction. For a system of 6 transactions the fraction of tasks with pessimism obtained by Nolin's method is over 20%, while for the mixed methods , this number does not exceed 4%.

The figure 9 shows that the time required by NM1 is almost the same as Nolin's method (while NM1 gives better results) and that the time required by NM2 is growing a little faster. As a conclusion, for systems of 6-10 transactions (which looks like an averagesis system), the best quality/time seems to be NM1 or NM2 since the cost in time is not high compared to Nolin's method, while the pessimism is significantly reduced. NM3 needs a significantly higher computing time for a low improvement compared to NM2.

6 Conclusion and Perspectives

For response time analysis of tasks with offset scheduled under fixed priority, only intractable techniques (exponential complexity) provide an exact evaluation of WCRT. Approximate techniques provide a pessimistic upper bounds for WCRT with a pseudo-polynomial complexity. In this article we have presented new WCRT analysis method that is a result of combination of the exact calculation and approximative calculation principles. This new method provides an upper bounds for WCRT with less pessimism, and it has a tunable pseudo polynomial complexity.

In our future works, we will use the monotonicity property of transaction and the tasks dominance property as a basement to introduce a new evaluation method in order to decrease the number of critical instants candidates taken into account.

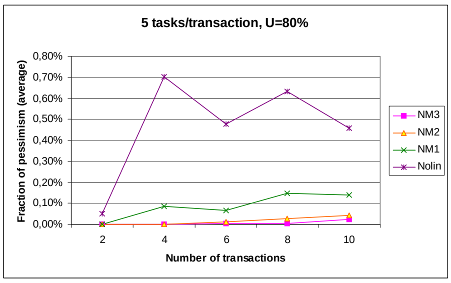

FIG. 6 – *Influence of the number of transactions on the pessimism.*

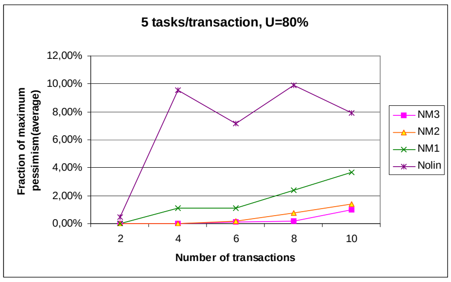

FIG. 7 – *Influence of the number of transactions on the maximum pessimism.*

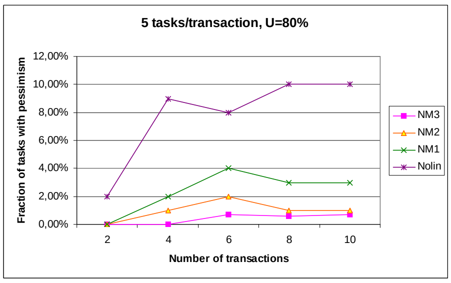

FIG. 8 – *Influence of the number of transactions on the tasks with pessimism.*

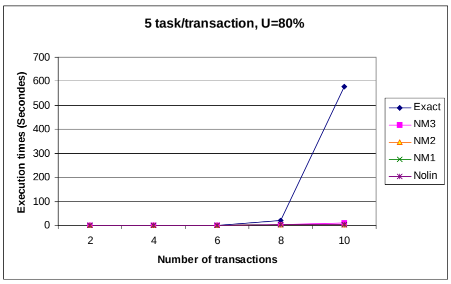

FIG. 9 – *Influence of the number of transactions on the execution times.*

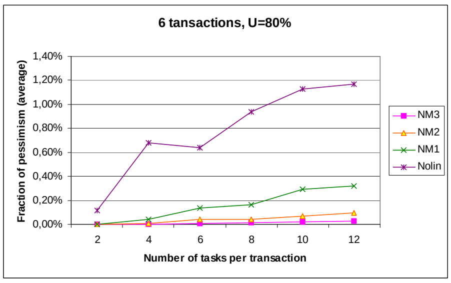

FIG. 10 – *Influence of the number of tasks on the pessimism.*

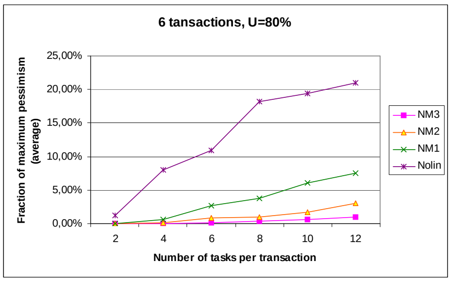

FIG. 11 – *Influence of the number of tasks on the maximum pessimism.*

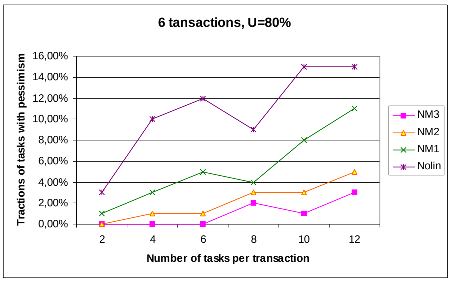

FIG. 12 – *Influence of the number of tasks on the tasks with pessimism.*

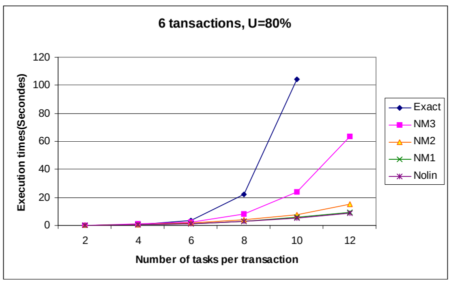

FIG. 13 – *Influence of number of tasks on the execution times.*

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