Robust state feedback admissibilization of discrete linear polytopic descriptor systems: A strict LMI Approach

Submitted to Automatica (in a shorter version)

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Version 7

Abstract

This paper deals with a new approach to the robust state feedback control of a class of discrete linear descriptor systems. A strict LMI (Linear Matrix Inequality)-based condition for the admissibility of a discrete descriptor model associated with matrix pencil (E,A) belonging to some set is proposed. The originality lies in the fact that the LMI is linear with respect to A and E. Therefore parameter deflections in the entries of A but also of E can be easily taken into account owing to polytopic description to derive an LMI sufficient robust admissibility condition. Moreover it is shown that the condition can be straightforwardly exploited to derive a static state feedback control law that admissibilizes a closed-loop uncertain descriptor model.

Keywords: Descriptor systems, robust admissibility, matrix inertia, strict LMI.

1 Introduction

This paper is concerned with the problem of admissibility and admissibilization of discrete singular systems $Ex_{k+1} = Ax_k$, (where E is possibly singular). The study of the singular system, roughly speaking, amounts to the study of the properties of the associated pencil (E, A). Singular (implicit, or descriptor) systems are of great interest as shown in the literature [1, 2, 3]. The descriptor form provides models that are more general and sometimes more natural than the conventional ones [3]. It includes information about static constraints on the state variables in addition to the dynamic ones. Thus, it is useful to represent mechanical systems [4], electrical networks [5], interconnected systems [6], and so on. The interested reader can pay attention to [2, 1, 7] and the references therein.

Many classical concepts and results obtained for conventional systems have been extended to descriptor ones. Let us quote, for instance, controllability and observability, pole assignment, stability analysis [1, 8] and stabilization techniques as well as disturbance rejection [9] and robustness aspects [10].

One of the main differences between conventional models ($E = I$) and implicit ones is that for implicit models, the roots of the pencil $(E,\ A)$ can be either finite or infinite. One is usually only interested in the clustering of finite roots in some specified region $\mathscr{D}$ of the complex plane. If $\mathscr{D}$ is the unit disc, this is referred to as the Schur-stability of the pencil (E,A) and it corresponds to the asymptotic stability of the associated discrete descriptor model. But infinite poles are also influent. In addition to (Schur)-stability, one has to check that the system is regular, meaning that there is a single solution to its state description (some kind of well-posedness), and that it is causal [1]. When the implicit model is (Schur)-stable, regular and causal, it is said to be (Schur)-admissible. In this paper, admissibility and admissibilization by static state feedback control law are addressed.

Among the most well-known simple conditions to test admissibility are the generalized Lyapunov equations [11, 3, 12, 8, 13]. However, the exploitation of such equations in an uncertain context or for design purpose is not always obvious. For this reason, other conditions expressed in terms of LMI (Linear Matrix Inequalities) are proposed [7]. If *strict* LMI are prefered, some of the first contributions are [14, 15, 16, 17].

Actually, many contributions deal with robust analysis or robust control of descriptor models [18], particularly through (not necessarily strict) LMI approach : see *e.g.* [10, 19],the seminal work of Masubuchi [20, 21] and the references therein. But very few really consider uncertainty on E. Let us quote [22] where A is however precisely known or [23] for interval matrices. In [24], the case of norm bounded Linear Fractional Transforms (LFT)-based uncertainties on A and E is addressed. Some very good insights can also be found in [25, 26].

Based on some properties of the inertia of Lyapunov-like matrices [27, 28], a *strict* LMI-formulation is adopted to express a sufficient condition for the admissibility of a discrete implicit system that belongs to a quite large class. The authors really insist on the strictness of the conditions from a computational point of view. Indeed, non strict inequalities are more difficult to handle and equalities impose constraints on the decision variables which make the optimization process harder to solve by existing software and often inappropriate in an uncertain context. It is emphasized that the obtained condition is straightforwardly extended to the case where both matrices A and E are subject to parameter uncertainties involved through a polytopic description. The idea is to prove the existence of a suitable "parameter-dependent Lyapunov-like" matrix, following arguments close to those originally proposed in [29].

Next, this condition is used to derive a state feedback control law that enables the designer to reach the nominal or robust closed-loop robust admissibility. To the authors' knowledge, it is the first time that the property of the inertia of the Lyapunov-like matrix is exploited for a design purpose.

This paper is organized as follows. Section 2 gives the problem formulation with preliminaries. Section 3 gives the LMI condition for Schur-admissibility of a discrete descriptor system (in the specified class). In section 4, the result is adapted to the presence of polytopic uncertainties affecting matrices A and E. The static state feedback problems for nominal (resp. robust) discrete linear descriptor systems is tackled in Section 5 (resp.Section 6). Section 7 provides a numerical illustration before to conlude.

Notation: In the remainder of the paper the notation is standard unless specified. M' is the transpose conjugate of M (only conjugate for scalars). I_q is the identity matrix of order q. I and 0 are identity and zero matrices of appropriate dimensions respectively. In matrix inequalities, $M > 0$ ($M < 0$) means that the matrix M is symmetric and positive (negative) definite. $\mathrm{Ker}(M)$ is the matrix the columns of which span the right nullspace of the subspace generated by the columns of M. $\mathrm{In}(M) = [n_+ \ n_- \ n_0]$ is the inertia of the square matrix M where n_+, n_- and n_0 are the number of positive, negative and zero eigenvalues respectively. In the sequel, $M^H = M + M'$ for any square matrix M. Symbol $\otimes$ denotes Kronecker's matrix product. At last $\mathrm{d}^\circ(P(z))$ is the degree of the polynomial P in the unknown z, $\mathbf{D}^-$ is the open unit disc and $\mathscr{C}$ is the unit circle.

2 Problem formulation

In this section, a series of definitions, properties, lemmas about matrix pencils and descriptor systems is poposed.

Consider the following discrete descriptor system:

$$E\, x_{k+1} = A\, x_k + B\, u_k, \tag{1}$$

where $x_k \in \mathbb{R}^n$ is the state vector and $u_k \in \mathbb{R}^m$ is the control vector. $\{A;E\} \in \{\mathbb{R}^{n\times n}\}^2$ are real square matrices. Matrix E may be singular. At this stage, the matrices are perfectly known. Some uncertainties in their entries will be introduced in Section 4. Several properties of this excited system can be studied through the autonomous system

$$E\, x_{k+1} = A\, x_k. \tag{2}$$

The distinction between the above descriptor model and the associated matrix linear pencil $(E,A) = E\lambda - A$ is usually omitted. Any mentioned property or definition is either the one of the pencil or the one of the associated system.

Definition 1 *Let $(E,A) = E\lambda - A$ be a linear matrix pencil where $\{A;E\} \in \{\mathbb{R}^{n\times n}\}^2$* (the notation (E,A) is usual but not universal) *and rank$(E) = r \leq n$. The* generalized spectrum *of the pencil (E,A) is denoted by $\lambda(E,A)$ and defined by*

$$\lambda(E,A) = \{\lambda \in (\mathbb{C} \cup \infty) : \det(E,A) = 0\}, \tag{3}$$

and its elements are referred to as the generalized eigenvalues *of (E,A).*
The finite generalized spectrum *$\lambda_f(E,A)$ is the subset of $(\lambda(E,A)$ that contains only the finite generalized eigenvalues.*
Moreover r is called the generalized order *of (E,A) and the* degree *f is the number of finite generalized eigenvalues, counting algebraic multiplicities.*

The spectrum $\lambda(E,A)$ contains p distinct finite elements with $p \leq f \leq r$ and thus the other ones are infinite. Obviously, $f = r = n$ when E is full rank and the pencil can describe a conventional linear time invariant system.

Definition 2 *[1] Let (E,A) be a linear matrix pencil where $\{A;E\} \in \{\mathbb{R}^{n\times n}\}^2$, with generalized order r and degree f. It is said to be:*

1. regular *if the polynomial in z defined by* $\det(zE - A)$ *is not identically zero;*

2. causal *if* $f = \mathrm{d}^\circ(\det(zE - A)) = r$;

3. Schur-stable *if* $\lambda_f(E,\, A) \subset \mathbf{D}^-$.

Note that the regularity of (E,A) corresponds to the well-posedness of the descriptor system (2) (uniqueness of the solution to the state equation), that the causality of (E,A) is actually the causality of the static subsystem called *backward subsystem* [7] and that the Schur-stability of (E,A) is nothing but the asymptotic stability of the (dynamic) *forward subsystem* [7]. *The well-posedness is implied by any of the two other properties*. Therefore the stability, which is of course compulsory, is however not sufficient and must be completed by causality. This leads to introduce the notion of admissibility.

Definition 3 *[1] The discrete singular system (2) is said to be* Schur-admissible *or simply* admissible *if (E,A) is Schur-stable and causal.*

Now, the reader is reminded of some equivalent forms of a pencil (E,A). First, it must be recalled that any pencil (NEM,NAM) where N and M are full rank square matrices is called an equivalent form [30], meaning that the associated system has the same properties as (2) (well-posedness, stability, causality). The first form is the so-called *SVD-like* form since it is based upon the singular value decomposition (SVD) of E. Indeed, for any pencil (E,A) with generalized order r, there exist two nonsingular matrices Q and T such that

$$\bar{A} = \begin{bmatrix} \bar{A}_{11} & \bar{A}_{12} \\ \bar{A}_{21} & \bar{A}_{22} \end{bmatrix} = QAT, \quad \bar{E} = \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix} = QET, \quad \text{with} \quad \bar{A}_{11} \in \mathbf{R}^{r\times r}. \tag{4}$$

Let $\mathbf{Q}$ and $\mathbf{W}$ be two non singular matrices such that the singular value decomposition of E is

$$\mathbf{Q}E\mathbf{W} = \begin{bmatrix} \Sigma & 0 \\ 0 & 0 \end{bmatrix} = \bar{E} \underbrace{\begin{bmatrix} \Sigma & 0 \\ 0 & I \end{bmatrix}}_{\mathbf{S}}, \tag{5}$$

where $\Sigma \in \mathbf{R}^{r\times r}$ is a diagonal matrix the diagonal entries of which are the non zero singular values of E. Clearly, it comes

$$\bar{E} = \mathbf{Q}E\mathbf{W}\mathbf{S}^{-1} = \mathbf{Q}ET. \tag{6}$$

Let the matrix $\breve{A}$ be defined by

$$\bar{A} = \mathbf{Q}AT. \tag{7}$$

The pencil $(\bar{E},\bar{A})$ is an SVD-like form equivalent to (E,A). Let $\mathbf{M}$ be defined by

$$\mathbf{M} = \mathbf{Q}^{-1} = \begin{bmatrix} \mathbf{M}_1 & \mathbf{M}_2 \\ \mathbf{M}_3 & \mathbf{M}_4 \end{bmatrix}, \qquad \mathbf{M}_1 \in \mathbf{R}^{r\times r}. \tag{8}$$

Also let the matrix $R \in \mathbf{R}^{r\times q}$ with $q = n - r$, be such that

$$\mathrm{Ker}\left(\begin{bmatrix} \mathbf{M}_1 & \mathbf{M}_2 \end{bmatrix}\right) = \begin{bmatrix} -R \\ I_q \end{bmatrix}, \tag{9}$$

from which the following nonsingular matrix is defined,

$$\mathbf{J} = \begin{bmatrix} I & -R \\ 0 & I \end{bmatrix} \Rightarrow \mathbf{J}^{-1} = \begin{bmatrix} I & R \\ 0 & I \end{bmatrix}. \tag{10}$$

Since $\mathbf{J}^{-1}\bar{E} = \bar{E}$, one can easily (and with no loss of generality) substitute $\mathbf{Q}$ with $Q = \mathbf{J}^{-1}\mathbf{Q}$ in (7) and then recover (4). One can then also check that

$$M = Q^{-1} = \mathbf{Q}^{-1}J = \begin{bmatrix} M_1 & 0 \\ M_3 & M_4 \end{bmatrix}. \tag{11}$$

Besides, let $(E,\ A)$ be a *regular* pencil with degree f and generalized order r. There exist two nonsingular matrices U and V such that

$$\tilde{A} = \begin{bmatrix} \hat{A} & 0 \\ 0 & I \end{bmatrix} = UAV, \quad \tilde{E} = \begin{bmatrix} I_f & 0 \\ 0 & N \end{bmatrix} = UEV, \tag{12}$$

where the eigenvalues of $\hat{A} \in \mathbb{R}^{f\times f}$ are the finite generalized eigenvalues of $(E,\ A)$ and where $N \in \mathbb{C}^{(n-f)\times(n-f)}$ is a nilpotent matrix which actually equals 0 when $f = r$. Such an equivalent form is referred to as the *Weierstraß* form of the pencil.

It must be recalled that the causality of the associated descriptor model is equivalent to $f = r$, but also to the nonsingularity of $\bar{A}_{22}$ [1]. In this case, the Weierstraß form becomes a special instance of the SVD-like form. It is then possible to get a Weierstraß form from any SVD-like form through the equivalence operation

$$\tilde{A} = \Gamma\bar{A}\Upsilon = \begin{bmatrix} \hat{A} & 0 \\ 0 & I_{n-r} \end{bmatrix}, \quad \tilde{E} = \Gamma\bar{E}\Upsilon = \bar{E}, \tag{13}$$

with

$$\hat{A} = \bar{A}_{11} - \bar{A}_{12}\bar{A}_{22}^{-1}\bar{A}_{21},\ \Gamma = \begin{bmatrix} I_r & -\bar{A}_{12}\bar{A}_{22}^{-1} \\ 0 & I_{n-r} \end{bmatrix} \quad \text{and} \quad \Upsilon = \begin{bmatrix} I_r & 0 \\ -\bar{A}_{22}^{-1}\bar{A}_{21} & \bar{A}_{22}^{-1} \end{bmatrix}, \tag{14}$$

meaning that $U = \Gamma Q$ and $V = T\Upsilon$.

In the remainder of the paper, we shall only consider pencils that belong to the set $\mathbb{S} = \mathbb{S}_1 \cup \mathbb{S}_2$ where:

- The subset $\mathbb{S}_1$ contains the pencils (E,A) for which $M_3 = 0$ in (11).
- the subset $\mathbb{S}_2$ contains the pencils (E,A) that lead to $\bar{A}_{12} = 0$ in (4) with (11).

In practice, the set $\mathbb{S}$ is far from empty and many applications lead to models in $\mathbb{S}$. A discussion about the relevance of this set is provided in appendix.

The purpose is to propose LMI conditions to, first, analyze the admissibility of system (2), second, admissibilize the system (1) by a static state feedback control law, third, take polytopic uncertainties on matrices A, B, and E into account in both analysis and design. To reach this goal, we propose a Lyapunov-like admissibility condition of a nominal implicit model as in (2). The main advantages of this condition are as follows:

- There is one single inequality unlike many contributions in the literature;
- The equivalence transformations are not explicitly required in the condition unlike some other contributions;
- The LMI is strict.

Nevertheless, the drawback is that it is proved to be necessary only is some special (but numerous) cases.
Moreover, from this condition, design techniques are derived, both in the nominal and robust cases. Then two other advantages of the condition are highlighted:

- It is straightforwardly extended to the state feedback design problem;
- The polytopic uncertainty on A and E can be naturally involved.

Remark 4 *The question of the strictness of the LMI might not seem crucial. Indeed, many LMI conditions encountered in the realm of descriptor systems involve a condition of the form [21, 7]*

$$X'E' = EX \geq 0, \tag{15}$$

or of the form [31, 8, 32]

$$E'XE \geq 0. \tag{16}$$

Condition (15) consists in both an equality and an inequality. Although such kinds of equalities can usually be taken into account by LMI solvers, the numerical experiments tend to show that it might be preferable to avoid them. But, for both conditions (15) and (16), the problem is mostly due to the non strict inequality. As pointed out in [7], the reliability of the numerical tools is rather limited when facing those kinds of inequalities. Particulary, it is sometimes difficult to distinguish infeasibility and marginal feasibility. Nevertheless, it is also explained, for example in [7], how these large inequalities can be transformed into strict inequalities by introducing additonal variables. Unfortunately, ***those transformations can no longer be efficiently exploited when E is uncertain.*** *This comes from the fact that they require the computation of a* constant *matrix $L = Ker(E)$, like in the paper [10]. It means that most of the existing LMI conditions are restricted to descriptor models with a precisely known matrix E. In the same way, [17] proposes an alternative to transform non strict LMI into strict ones. But $Ker(E)$ and its complement also need to be computed, what is not suitable in the presence of polytopic uncertainties on E. More precisely, in [17, Theorem 2], robust stability against descriptor LFT-based uncertainties on both A and E is addressed through strict inequalities. But it is unclear how to deal with causality and the structure of the proposed* "separator" *is hardly exploitable in a polytopic context. However, there are many examples where a polytopic uncertainty on E is revealed as very useful, even for some conventional systems for which an implicit description makes the uncertainty structure simpler.*

3 Admissibility of discrete descriptor systems

In this section, an LMI condition for Schur-admissibility of a system in $\mathbb{S}$ is established. But before, it is required to state a proposition related to the so-called Generalized Discrete Lyapunov Inequality.

3.1 Generalized Discrete Lyapunov Inequality (GDLI)

In this subsection, a preliminary theorem is presented, which is very close to well-known results concerning the admissibility of a discrete linear implicit model. However, the difference, though maybe weak, introduces special insights of importance for our purpose.

Theorem 5 *Let a discrete implicit system be described by (2) with generalized order r. It is admissible if and only if there exists a non singular symmetric matrix P with inertia In(P)* $= [r\ (n-r)\ 0]$ *such that*

$$-EPE' + APA' < 0. \tag{17}$$

In this event, any solution to (17) has the same inertia.

Proof. First, it is proved that there exists $P = P'$ solution to (17) if and only if (E,A) has no generalized eigenvalue on the unit circle $\mathscr{C}$. It is what is called the $\mathscr{C}$-regularity of the pencil, this expression being borrowed from [28]. The inertia in case of admissibility will be discussed after. Let $\mathscr{C}$ be the unit circle defined by

$$\mathscr{C} = \left\{ s \in \mathbb{C} : \begin{bmatrix} s' \\ 1 \end{bmatrix}' R \begin{bmatrix} s' \\ 1 \end{bmatrix} = 0 \right\}, \tag{18}$$

with $R = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$. The pencil (E,A) is $\mathscr{C}$-regular if and only if

$$\det(sE - A) \neq 0 \quad \forall s \in \mathscr{C} \tag{19}$$

$$\Leftrightarrow (sE-A)(-\tau I)(sE-A)' < 0 \; \forall s \in \mathscr{C}, \forall \tau > 0 \tag{20}$$

$$\Leftrightarrow \begin{bmatrix} sE' \\ I \end{bmatrix}' \underbrace{\begin{bmatrix} I \\ -A \end{bmatrix} (-\tau I) \begin{bmatrix} I \\ -A \end{bmatrix}'}_{\Theta} \underbrace{\begin{bmatrix} sE' \\ I \end{bmatrix}}_{N_\lambda} < 0 \; \forall s \in \mathscr{C}, \forall \tau > 0. \tag{21}$$

If one defines the following matrices,

$$\Gamma_\lambda(s) = \begin{bmatrix} I & -s'I \end{bmatrix}, \quad X_\lambda = \begin{bmatrix} I & 0 \\ 0 & E' \end{bmatrix}, \tag{22}$$

one can then notice that

$$N_\lambda = \mathrm{Ker}(\Gamma_\lambda(s) X_\lambda) \quad \forall s \in \mathscr{C}. \tag{23}$$

Thus, by virtue of the full block S-procedure [33, Theorem 8], inequality (21) holds if and only if there exists a nonsingular matrix $P = P'$ such that

$$X_\lambda' \begin{bmatrix} P & 0 \\ 0 & -P \end{bmatrix} X_\lambda + \Theta < 0 \quad \forall \tau > 0, \tag{24}$$

which can be written

$$\begin{bmatrix} I & 0 \\ 0 & E \end{bmatrix} \begin{bmatrix} P & 0 \\ 0 & -P \end{bmatrix} \begin{bmatrix} I & 0 \\ 0 & E' \end{bmatrix} + \Theta < 0 \quad \forall \tau > 0. \tag{25}$$

By using Finsler's lemma [34, Theorem 2.3.10], condition (25) is equivalent to (17) which is some extended Lyapunov-like inequality for descriptor models.

The following reasoning concerns the inertia of matrix P. Like for Lyapunov's or Stein's inequality applied to conventional non descriptor models (see [28] and the references therein), the inertia of matrix P depends on the distribution of the eigenvalues with respect to $\mathscr{C}$. So assume that (E,A) is

Schur-admissible and that there exists a matrix P solution to (17). Using the Weierstraß form (13) one gets

$$U^{-1}\tilde{A}\underbrace{V^{-1}P(V')^{-1}}_{W}\tilde{A}(U')^{-1}-U^{-1}\tilde{E}\underbrace{V^{-1}P(V')^{-1}}_{W}\tilde{E}'(U')^{-1}<0, \tag{26}$$

$$\Leftrightarrow \tilde{A}W\tilde{A}'-\tilde{E}W\tilde{E}'<0, \text{ with } W=\left[\begin{array}{cc} W_1 & W_3 \\ W_3' & W_2 \end{array}\right], \quad W_1\in\mathbb{R}^{r\times r}. \tag{27}$$

Besides, inequality (27) can be written

$$\left[\begin{array}{c|c} -W_1+\hat{A}W_1\hat{A}' & \hat{A}W_3 \\ \hline W_3'\hat{A}' & W_2 \end{array}\right]<0. \tag{28}$$

Since $\hat{A}$ is Schur-stable, the (1,1)-block of (28) shows that $W_1>0$ by virtue of Stein's theorem [35]. From the (2,2)-part of of (28), $W_2<0$. $W_1>0$ implies that the r first columns of W, span a subspace associated with r positive eigenvalues. $W_2>0$ implies that the last $(n-r)$ columns span a subspace associated with $(n-r)$ negative eigenvalues. So $\mathrm{In}(P)=[r\ (n-r)\ 0]$. □

Remark 6 *It must be noticed that some results on the inertias of solutions to generalized Lyapunov equations related to descriptor models exist (see [27] with the references therein) and the present contribution is of course coherent with this literature,* though different. *It matters to be aware that Lyapunov's inequality has* always *a solution unlike Lyapunov's equality which might have none when E is singular. Hence, one of the authors' motivations to clearly introduce Theorem 5. The designer can exploit this property of the inertia of P for analysis and design purpose in a polytopic context as it will be seen in the following sections.*
Besides, inequality (17) is already known [32, 7]. But it is accompanied by the constraint $EPE'\geq 0$ which the authors wanted to avoid due to forthcoming polytopic uncertainties. So to invoke [32] would have only proved the necessity of (17) and In(P) would not have been completely deduced, although it is crucial for our purpose.

3.2 An LMI condition for admissibility

Based on the previous result, we now focus on the admissibility analysis through a *strict* LMI that is *linear with respect to A and E*.

Theorem 7 *Let the descriptor model be described by (2) and belong to $\mathbb{S}$; It is Schur-admissible if and only if there exist matrices $Y=Y'$ and G such that*

$$L=\left[\begin{array}{cc} -Y & 0 \\ 0 & Y \end{array}\right]+\left\{\left[\begin{array}{c} A \\ -E \end{array}\right]G\left[\begin{array}{cccc} 0 & 0 & I_r & 0 \\ 0 & I_{n-r} & 0 & 0 \end{array}\right]\right\}^H<0. \tag{29}$$

Moreover, the matrix G is necessarily nonsingular.

Proof.
Sufficiency: First, let Y be divided into four blocks as follows,

$$Y=\left[\begin{array}{cc} Y_1 & Y_3 \\ Y_3' & Y_2 \end{array}\right],$$

where $Y_1 \in \mathbb{R}^{r\times r}$. The $(r\times r)$-north-western block of the left handside member of condition (29) shows that $-Y_1 < 0 \Leftrightarrow Y_1 > 0$. Moreover, the $(q\times q)$-south-eastern block of the inequality shows that $Y_2 < 0$ (with $q = n-r$). So, one has

$$\left\{\begin{array}{l} Y_1 > 0, \\ Y_2 < 0, \end{array}\right. . \tag{30}$$

In the same fashion as in the proof of Theorem 5, it can be proved that

$$\mathrm{In}(Y) = [r \ \ q \ \ 0]. \tag{31}$$

Besides, by using transformation (12) and by virtue of Sylvester's theorem (*i.e.* by congruence), on can deduce from (29) that

$$(I_2 \otimes U)L(I_2 \otimes U') = \left[\begin{array}{cc} -\tilde{P} & 0 \\ 0 & \tilde{P} \end{array}\right] + \left\{\left[\begin{array}{c} \tilde{A} \\ -\tilde{E} \end{array}\right]\tilde{D}\right\}^H < 0, \tag{32}$$

with $\tilde{P} = UYU'$ and

$$\tilde{D} = V^{-1}G\left[\begin{array}{cccc} 0 & 0 & I_r & 0 \\ 0 & I_{n-r} & 0 & 0 \end{array}\right](I_2 \otimes U'). \tag{33}$$

Now define matrix $\tilde{Z}$ by

$$\tilde{Z} = \left[\begin{array}{cc} \tilde{E} & \tilde{A} \end{array}\right]. \tag{34}$$

It is clear that

$$\left[\begin{array}{c} \tilde{A} \\ -\tilde{E} \end{array}\right] = \mathrm{Ker}(\tilde{Z}). \tag{35}$$

Therefore, left and right multiply inequality (32) by $\tilde{Z}$ and $\tilde{Z}'$ respectively to get

$$\tilde{A}\tilde{P}\tilde{A}' - \tilde{E}\tilde{P}\tilde{E}' < 0. \tag{36}$$

Using transformation (12) again, (36) can be written

$$UA\underbrace{V\tilde{P}V'}_{P}A'U' - UE\underbrace{V\tilde{P}V'}_{P}E'U' < 0, \tag{37}$$

which, once more by congruence, is equivalent to (17). Since $\mathrm{In}(P) = \mathrm{In}(\tilde{P}) = [r\ q\ 0]$ (Sylvester's theorem), by virtue of Theorem 5, the pencil (E,A) is Schur-admissible.

Necessity: Assume that $(E,A) \in \mathbb{S}$ is Schur-admissible, then so are $(\bar{E},\bar{A})$, the equivalent form highlighted in (4), as well as the Weierstraß form $(\tilde{E},\tilde{A})$ given in (13-14). Matrix $\hat{A}$ is Schur-stable so by Stein's theorem [35], one can find a matrix $\hat{P} = \hat{P}' > 0$ such that

$$-\hat{P} + \hat{A}\hat{P}\hat{A}' < 0. \tag{38}$$

Consider the largest positive scalar number μ such that

$$-\hat{P} + \hat{A}\hat{P}\hat{A}' + \mu\bar{A}_{12}\bar{A}_{12}' < 0. \tag{39}$$

Define the following matrices:

$$\mathbf{B} = \begin{bmatrix} 0 & 0 & I_r & 0 \\ 0 & I_q & 0 & 0 \end{bmatrix}', \quad \mathbf{C} = \begin{bmatrix} \bar{A}' & -\bar{E}' \end{bmatrix} = \begin{bmatrix} \bar{A}'_{11} & \bar{A}'_{21} & -I_r & 0 \\ \bar{A}'_{12} & \bar{A}'_{22} & 0 & 0 \end{bmatrix}. \tag{40}$$

Also notice that, since $(\bar{E},\bar{A})$ is causal, $\bar{A}_{22}^{-1}$ exists and one can easily give the following expressions:

$$\mathrm{Ker}(\mathbf{B}') = \begin{bmatrix} I_r & 0 & 0 & 0 \\ 0 & 0 & 0 & I_q \end{bmatrix}', \quad \mathrm{Ker}(\mathbf{C}) = \begin{bmatrix} I & -\bar{A}_{12}\bar{A}_{22}^{-1} & \hat{A} & 0 \\ 0 & 0 & 0 & I_q \end{bmatrix}'. \tag{41}$$

It can be checked that, with

$$\Phi = \begin{bmatrix} -\hat{P} & 0 & 0 & 0 \\ 0 & \mu\bar{A}_{22}\bar{A}'_{22} & 0 & 0 \\ 0 & 0 & \hat{P} & 0 \\ 0 & 0 & 0 & -\mu\bar{A}_{22}\bar{A}'_{22} \end{bmatrix}, \tag{42}$$

due to (39), one gets the two inequalities given in (66). So by virtue of Lemma 12in appendix, there exists a matrix $\bar{\mathscr{G}}$ such that (67) holds, or equivalently,

$$\Leftrightarrow \begin{bmatrix} -\bar{Y} & 0 \\ 0 & \bar{Y} \end{bmatrix} + \left\{ \begin{bmatrix} \bar{A} \\ -\bar{E} \end{bmatrix} \bar{D} \right\}^H < 0, \tag{43}$$

with

$$\bar{Y} = \begin{bmatrix} \hat{P} & 0 \\ 0 & -\mu\bar{A}_{22}\bar{A}'_{22} \end{bmatrix}, \quad \bar{D} = -\mathbf{C}'\bar{\mathscr{G}}'\mathbf{B}'. \tag{44}$$

Now, from the equivalence (4), inequality (43) amounts to

$$\begin{bmatrix} -\bar{Y} & 0 \\ 0 & \bar{Y} \end{bmatrix} + \left\{ \begin{bmatrix} QA \\ -QE \end{bmatrix} T\bar{D} \right\}^H < 0 \tag{45}$$

$$\Leftrightarrow \underbrace{\begin{bmatrix} -Y & 0 \\ 0 & Y \end{bmatrix}}_{\Psi} + \left\{ \begin{bmatrix} A \\ -E \end{bmatrix} D \right\}^H < 0, \tag{46}$$

with $Y = M\bar{Y}M'$ (where M equals Q^{-1}) and

$$D = T\bar{D}(I_2 \otimes (M')). \tag{47}$$

Now, define Z as a matrix such that

$$\mathrm{Ker}(Z) = \begin{bmatrix} A \\ -E \end{bmatrix}. \tag{48}$$

Then, it is easy to deduce, by left and right multiplying (46) by Z and Z' respectively, that, on the one hand,

$$Z\Psi Z' < 0. \tag{49}$$

On the other hand, because of the particular structure of M given in (11), the matrix Y complies with

$$Y = M\bar{Y}M' = \begin{bmatrix} Y_1 & Y_3 \\ Y'_3 & Y_2 \end{bmatrix}, \tag{50}$$

where

$$Y_1 = M_1 \hat{P} M_1' > 0 \quad \text{and} \quad Y_2 = M_3 \hat{P} M_3' - \mu M_4 \bar{A}_{22} \bar{A}_{22}' M_4'. \tag{51}$$

Since M_1, M_4 and $\bar{A}_{22}$ are full rank matrices, then with a sufficiently large μ, one gets $Y_2 < 0$. But μ is not arbitrary in the general case. However, if (E,A) belongs to $\mathbb{S}_2$, then $\bar{A}_{12} = 0$ thus it is clear from (39) that μ can be chosen arbitrarily large *i.e.* large enough to get $Y_2 < 0$. Besides, if (E,A) belong to $\mathbb{S}_1$, then $M_3 = 0$ what straightforwadly implies $Y_2 < 0$. Therefore, if (E,A) belongs to $\mathbb{S} = \mathbb{S}_1 \cup \mathbb{S}_2$, then

$$\text{Ker}(\mathbf{B}')' \Psi \text{Ker}(\mathbf{B}') < 0. \tag{52}$$

By virtue of Lemma 12 again, (49) together with (52) are equivalent to the existence of some matrix G such that (29) holds.
It is now proved that G is invertible. Assume that there exists a singular matrix G such that (29) holds. There exists a non zero vector $z \in \mathbb{R}^n$ such that $Gz = 0$. Left and right multiply inequality (62) by

$$z' \begin{bmatrix} 0 & 0 & I_r & 0 \\ 0 & I_{n-r} & 0 & 0 \end{bmatrix}$$

and its tranpose respectively. Then, it comes

$$z' \begin{bmatrix} Y_1 & 0 \\ 0 & -Y_2 \end{bmatrix} z < 0 \tag{53}$$

which is clearly impossible since $Y_1 > 0$ and $Y_2 < 0$. $\square$

There are several strict LMI conditions for the Schur-admissibility of a pencil in the literature, such as [14, 15, 36]. Those conditions are necessary even when $(E,A) \in \mathbb{S}$. Unfortunately, they do not easily enable one to take polytopic uncertainty into account.

Even if $(E,A) \in \mathbb{S}^-$, it is possible that the condition is feasible. However, counterexamples can be found so (29) is not necessary in the general case.

4 Robust admissibility

In this section, the robust analysis of a discrete implicit linear system containing uncertainties is addressed. The condition (29) is extended to the case where the matrices A and E are subject to a polytopic uncertainty. Define the simplex

$$\Omega = \left\{ \alpha = \begin{bmatrix} \alpha_1 \\ \vdots \\ \alpha_N \end{bmatrix} \in \mathbb{R}^n, \text{with}\, \alpha_i \geq 0 \forall i \in \{1, \ldots, N\}, \sum_{i=1}^{N} \alpha_i = 1 \right\}. \tag{54}$$

The uncertain descriptor model is defined by (1) but with an uncertain triplet of matrices $(A,\ E,\ B)$ that complies with

$$\begin{cases} M(\alpha) = [E\ A\ B] = \sum_{i=1}^{N} \alpha_i M_i = \sum_{i=1}^{N} \alpha_i \left[E_i\ A_i\ B_i\right], & \\ \text{rank}(E(\alpha)) = r & \forall \alpha \in \Omega, \\ (E(\alpha), A(\alpha)) \in \mathbb{S} & \forall \alpha \in \Omega. \end{cases} \tag{55}$$

The extreme matrices M_i are the vertices of a polytope in which $M(\alpha)$ lies when α describes Ω. It is a rather classical description of uncertainty that is exploited in practice to handle affine dependency on parameters. Note however that there is here a restriction since the rank of E is assumed to be constant over Ω. It is not a drastic restriction since, in practice, the rank is directly related to the structure of the system (*i.e.* to the number of dynamic and static constraints) that does not change in the presence of uncertainty. This assumption is to be connected to that in [25]. It is hard to check by computational techniques applied to the polytopic pencil but can very often be simply verified on the original affinely parameter-dependent model with no numerical tool.
The constraint related to $\mathbb{S}$ is discussed in appendix.

The authors insist on the fact that the polytopic uncertainty also affects matrix E which is one of the main motivations in this work. Descriptor forms are meant to arise quite naturally when describing the behaviour of the systems. It is particularly true when parameter deflections are expected. A typical example is an uncertainty on some inertias in the mechanical devices. In such a case, it really matters to preserve the possible linear involvement of the parameters that can appear on E.

The problem for a system corrupted by uncertainty is of course to preserve its performances for all admissible uncertainties or in other words for every instance of matrices $E(\alpha)$, $A(\alpha)$ and $B(\alpha)$, as expressed in the next definition.

Definition 8 *The uncertain discrete descriptor system (2) or (1) with (54-55) is robustly Schur-admissible, if it is Schur-admissible for any $\alpha \in \Omega$.*

Theorem 9 *The uncertain discrete descriptor system (2) or (1) with (54-55) is robustly Schur-admissible if there exist matrices $Y_i = Y_i' \in \mathbb{R}^{n\times n}$, $i = 1,\dots N$, and $G \in \mathbb{R}^{n\times n}$ such that*

$$\mathscr{F}_i = \begin{bmatrix} -Y_i & 0 \\ 0 & Y_i \end{bmatrix} + \left(\begin{bmatrix} A_i \\ -E_i \end{bmatrix} G \begin{bmatrix} 0 & 0 & I_r & 0 \\ 0 & I_{n-r} & 0 & 0 \end{bmatrix} \right)^H < 0 \quad \forall i \in \{1,\dots,N\} \tag{56}$$

Proof. First consider the following convex combination:

$$\mathscr{F} = \mathscr{F}(\alpha) = \sum_{i=1}^{N} \alpha_i \mathscr{F}_i < 0. \tag{57}$$

$$\Leftrightarrow \mathscr{F}(\alpha) = \begin{bmatrix} -Y(\alpha) & 0 \\ 0 & Y(\alpha) \end{bmatrix} + \left\{ \begin{bmatrix} A(\alpha) \\ -E(\alpha) \end{bmatrix} G \begin{bmatrix} 0_r & 0 & I_r & 0 \\ 0 & I_{n-r} & 0 & 0_{n-r} \end{bmatrix} \right\}^H < 0, \tag{58}$$

with

$$Y(\alpha) = \sum_{i=1}^{N} \alpha_i Y_i. \tag{59}$$

It is clear that the above inequality is nothing but (29) applied to pencil $(E(\alpha), A(\alpha))$, what proves the robust Schur-admissibility. $\square$

The linearity of condition (56) with respect to both A_i and E_i enables one to build the convex combination $\mathscr{F}(\alpha)$ and thus to make the pencil $(E(\alpha), A(\alpha))$ be straightforwardly involved in the inequality. This is, to the authors's knowledge, the first strict LMI condition that allows to consider a polytopic uncertainty on both matrices A and E. The idea is to make Y depend on α in a polytopic way by keeping G constant over the polytope rather than Y is inspired from the idea exploited in [29].

5 Admissibilization of discrete descriptor systems

In this section, an LMI condition to design a Schur-admissibilizing state feedback control law for (1) is proposed. So, it is aimed at finding a control law

$$u_k = Kx_k, \tag{60}$$

where the feedback matrix K is computed to reach the Schur-admissibility of

$$Ex_{k+1} = (A+BK)x_k. \tag{61}$$

Theorem 10 *The discrete linear implicit system (1) is Schur-admissibilizable by a state feedback control obeying to (60) if (and only if when $(E,A)\in\mathbb{S}_1$) there exist matrices $Y = Y' \in \mathbb{R}^{n\times n}$, $G \in \mathbb{R}^{n\times n}$ and $S \in \mathbb{R}^{m\times n}$ such that*

$$\begin{bmatrix} -Y & 0 \\ 0 & Y \end{bmatrix} + \left(\left(\begin{bmatrix} A \\ -E \end{bmatrix} G + \begin{bmatrix} B \\ 0 \end{bmatrix} S \right) \begin{bmatrix} 0 & 0 & I_r & 0 \\ 0 & I_{n-r} & 0 & 0 \end{bmatrix} \right)^H < 0. \tag{62}$$

In this event, a static state feedback control matrix is given by

$$K = SG^{-1}. \tag{63}$$

Proof. It suffices to use (63) *i.e.* to replace S by KG in (62). This change of variable yields condition (29) but with $A_c = A+BK$ replacing A, *i.e.*

$$\begin{bmatrix} -Y & 0 \\ 0 & Y \end{bmatrix} + \left\{ \begin{bmatrix} A_c \\ -E \end{bmatrix} G \begin{bmatrix} 0_r & 0 & I_r & 0 \\ 0 & I_{n-r} & 0 & 0 \end{bmatrix} \right\}^H < 0. \tag{64}$$

Therefore, by virtue of Theorem 7, the closed-loop descriptor system (61) is Schur-admissible. The non singularity of G is also guaranteed by Theorem 7. Necessity only holds for $(E,A)\in\mathbb{S}_1$ since $(E,A)\in\mathbb{S}_2$ does not imply $(E,A_c)\in\mathbb{S}_2$ □

6 Robust admissibilization

In this section, the feedback admissibilization problem for descriptor systems is still adressed, but for uncertain models. The condition (62) is extended to the case where the matrices involved in (61) are subject to a polytopic uncertainty.

Theorem 11 *The uncertain discrete implicit system (1) with (54-55) is robustly Schur-admissibilizable by a state feedback control obeying to (60) if there exist matrices $Y_i = Y_i' \in \mathbb{R}^{n\times n}$, $i = 1,\ldots,N$, $G \in \mathbb{R}^{n\times n}$, and $S \in \mathbb{R}^{m\times n}$ such that*

$$\mathscr{R}_i = \begin{bmatrix} -Y_i & 0 \\ 0 & Y_i \end{bmatrix} + \left(\left(\begin{bmatrix} A_i \\ -E_i \end{bmatrix} G + \begin{bmatrix} B_i \\ 0 \end{bmatrix} S \right) \begin{bmatrix} 0 & 0 & I_r & 0 \\ 0 & I_{n-r} & 0 & 0 \end{bmatrix} \right)^H < 0 \quad \forall i. \tag{65}$$

In this event, a static state feedback matrix is given by (63).

Proof. It is quite straighforward since it hinges upon the proofs of Theorems 9 and 10. It suffices to build the (obviously negative definite) convex combination $\mathscr{R}(\alpha) = \sum_{i=1}^{N} \alpha_i \mathscr{R}_i$ and then to apply the change of variable (63) in the sense $S = KG$ (note that, once again, G is nonsingular from Theorem 7) to recover the LMI admissibility condition applied to the uncertain closed-loop pencil $(E(\alpha), A_c(\alpha) = A(\alpha)+B(\alpha)K)$. A key point is that the obtained condition is also linear with respect to B_i. □

7 Illustrative example

In this section, the presented approach is illustrated by a numerical example. It is focused on admissibilization of a non causal (even non regular) singular system. Consider the descriptor model (1) with

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -20+\delta_2 & -20+\delta_2 & 2 & 2 \\ 2 & 0 & 0 & 0 \\ 5 & 2 & 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 & 1 \\ 1+0.5\delta_2 & \\ 0 & 0 \\ 0 & 0 \end{bmatrix},$$

$$E = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 50+\delta_1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

with $-5 \leq \delta_1 \leq 5$ and $-6 \leq \delta_2 \leq 6$. Note that $\mathrm{rank}(E)$ remains equal to 2 over the uncertainty domain. The pencil (E,A) belongs to $\mathbb{S}_2$ since it is a SVD-like form. However, it is not regular ($\det(sE-A) = 0\,\forall s$) and therefore neither stable nor causal. When δ_1 and δ_2 vary in their ranges then Matrix $[E\ A\ B]$ lies in a polytope of $N = 4$ vertices, as defined in (55), with (54).
The purpose is here to derive a static state feedback control law as given in (60), that makes the closed-loop system associated with pencil $(E,\ A+BK)$ be robustly admissible with respect to the polytope of matrices. An LMI solver is used to apply Theorem 11 and thus to find the feedback matrix

$$K = \begin{bmatrix} 29.4129 & 28.2683 & -2.6173 & -3.1251 \\ -0.0639 & -1.0642 & 0.0046 & 0.0071 \end{bmatrix}.$$

(The digits are meaningful but no additional are required.) This static state feedback robustly Schur-admissibilizes the considered descriptor system, *i.e.* the closed-loop system becomes regular, causal and Schur-stable for any instance of δ_1 and δ_2. The inertia of $Y(\alpha)$ is imposed: $\mathrm{In}(Y(\alpha)) = [r\ 2\ 0]$, meaning that any closed-loop system $(E(\alpha),\ A(\alpha)+B(\alpha)K)$, $\alpha \in \Omega$, has exactly $r = 2$ eigenvalues inside the unit disc, none on $\mathscr{C}$ and 2 at infinity.

8 Conclusions

In this paper, a matrix inequality that provides a new sufficient strict LMI condition for Schur-admissibility of discrete descriptor systems has been proposed. Based on this result, a simple and tractable strict LMI condition to synthesize a state feedback control law that Schur-stabilizes the closed-loop system has been derived. The use of the notion of matrix inertia allows the choice of the distribution of eigenvalues with respect to the unit circle. This has actually been used to guarantee the Schur-admissibility of the closed-loop singular system. Although the condition is only proved to be sufficient, it is at least necessary for a wide set og interesting models. Moreover, a robust Schur-admissibilizing state feedback control law has also been proposed for polytopic uncertain discrete singular systems. The presented numerical examples have illustrated the usefulness of the proposed methods. The authors insist on the strictness of the LMI condition and on its linearity with respect to E, A, and B. This linearity enables the consideration of uncertain parameters on singular matrix E, which, to the authors' knowledge, is rarely encountered in the literature. For this reason, it can hardly be compared to other conditions in the literature, in terms of conservatism.

Appendix

Lemma 12 (Simplified version of [34, Theorem 2.3.12]). *Let* Φ *be a symmetric matrix and let* $\mathbf{B}$ *and* $\mathbf{C}$ *be real matrices. The following statements are equivalent.*

i)

$$\begin{cases} \mathrm{Ker}(\mathbf{B}')'\Phi\mathrm{Ker}(\mathbf{B}') < 0, \\ \mathrm{Ker}(\mathbf{C})'\Phi\mathrm{Ker}(\mathbf{C}) < 0. \end{cases} \tag{66}$$

ii) There exists a matrix $\mathscr{G}$ *such that*

$$\Phi + (\mathbf{B}\mathscr{G}\mathbf{C})^H < 0. \tag{67}$$

Discussion about the relevance of the set $\mathbb{S}$

The set is essentially defined by $\mathbb{S}_1$. Firstly, any SVD-like form belongs to $\mathbb{S}_1$ (mechanical devices are naturally modelled by SVD forms). But it is also the case of any pencil with $E = \begin{bmatrix} E_{11} & E_{22} \\ 0 & 0 \end{bmatrix}$ and $E_{11} \in \mathbb{R}^{r\times r}$. Secondly, any pencil with $E = \begin{bmatrix} 0 & 0 \\ E_{21} & E_{22} \end{bmatrix}$ and $E_{22} \in \mathbb{R}^{r\times r}$ can be transformed into the previous one simply by appropriately sorting the state variables. Note that such a transformation modifies the location of the entries in A and E but not the entries themselves. Therefore, it preserves the polytopic structure of the uncertainty. Thirdly, once a nominal model is in $\mathbb{S}_1$, or can be easily transformed so as to be in, an affinely parameter-dependent uncertainty unlikely makes it leave $\mathbb{S}_1$, as illustrated by the example of section 7. At last, the set $\mathbb{S}_1$ is of course larger that the one simply made up by the above-mentioned cases. Examples tend to show that it is better to sort the state variables so as to have the r first rows of E linearly independent.

The set $\mathbb{S}_2$ is less easy to apprehend. It corresponds to a technical assumption. Although this set is clearly not empty, it is obviously smaller than $\mathbb{S}_1$ and its invariance against a polytopic uncertainty is difficult to check and actually not really probable. So, in practice, $\mathbb{S} \simeq \mathbb{S}_1$.

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