Robust $\mathcal{D}_{\mathbf{R}}$-stability analysis against a $\mathcal{JL}$-structured uncertainty

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Version 2

Index Terms

Robustness, $\mathcal{D}_R$-stability, $\mathcal{LMI}$

Abstract

This paper deals with the robust transient performances analysis of a linear state-space model through the concept of state matrix $\mathcal{D}_R$-stability (a matrix is $\mathcal{D}_R$-stable **if** *its spectrum is clustered in $\mathcal{D}_R$, a region of the complex plan). The considered uncertainty is additive $\mathcal{JL}$-structured. A robust $\mathcal{D}_R$-stability bound is proposed on the uncertain domain. The nominal matrix being $\mathcal{D}_R$-stable,* **if** *the robust $\mathcal{D}_R$-stability bound is not exceeded, then the uncertain matrix remains $\mathcal{D}_R$-stable. The bound is obtained by solving an $\mathcal{LMI}$ problem and reveals as not too pessimistic. The computation time is very short.*

1 Introduction

We are here interested in the robust analysis of the transient performances of a system. We consider a linear state-space system, expected to reach some performances with respect to its transient response (settling time, damping,...). We assume in addition that its state matrix $\mathbb{A}$ is subject to an additive $\mathcal{JL}$-structured uncertainty $\mathbb{E}$. The problem is transposed in the complex plane specified to guarantee some transient perfomances. The regions considered here are called $\mathcal{EMI}$-regions (Ellipsoidal Matrix Inequality [5]) and are generically denoted by $\mathcal{D}_R$. The clustering property of the spectrum is referred to as $\mathcal{D}_R$-stability of the matrix. $\mathcal{D}_R$ can be a vertical shifted half-plane in the case of a constraint over the settling time or a sector if a minimum damping must be guaranted, a disc, etc. The nominal $\mathcal{D}_R$-stability condition used in this work is the one proposed by [6]. The objective is to compute a bound ρ (to be defined) on uncertainty $\mathbb{E}$ so that the clustering condition previously quoted is always checked. Such a bound is known as a robust $\mathcal{D}_R$-stability bound.
Unfortunately, there is no method in the literature which really gives the not pessimistic ideal bound. This ideal bound is known as a $\mathcal{D}_R$-stability radius [8]. Among the various techniques allowing to approach this radius, one can quote the μ-analysis [3] because, for such a problem, μ, the structured

singular value, can be seen as the inverse of the stability radius. This technique is efficient because it makes possible, in particular, to take into account the reality of the uncertainty, distinguishing "μ-complex" from "μ-real", but presents also two disadvantages : a description of the problem that is not always obvious and a frequency scanning requiring an heavy computation. Other techniques lie on the use of generalized Lyapunov equations or on the formulation of $\mathcal{LMI}$-regions but these last are based on the use of triangular inequality inducing some degree of pessimism. This work constitues an attempt to propose a lower bound of the complex $\mathcal{D}_R$-stability radius which is not very conservative and which requires a weak computation time. The results are expressed in terms of $\mathcal{LMI}$.
The article is organized in the following way : in the second part, we will state the precise problem, recalling the definition of $\mathcal{D}_R$-stability and $\mathcal{EMI}$-regions, and then, by introducing the modeling of the $\mathcal{JL}$-structured uncertainty. It will finally enable us to clearly define our objectives. The third part will present the new computation technique of a robust $\mathcal{D}_R$ -stability bound whereas the fourth one part is devoted to numerical illustrations being based on models borrowed from the literature which will make possible to validate this new technique. We will conclude in the fifth part.
Notations : We denote by M', the transpose of M, by $\mathcal{S}(M)$ symmetric matrix $M + M'$ and by $\tilde{z}$ the conjugate of the scalar number z. The Kronecker product is denoted by $\otimes$. $||M||_2$ is the 2-norm of matrix M induced by the Euclidean vector-norm, *i.e.* the maximal singular value of M. $\lambda_i(M)$ corresponds to the i^{th} eigenvalue of matrix M and $\overline{\sigma}(M)$ denotes the maximal eigenvalue of M. $\mathbb{I}_n$ is the identity matrix of order n and $\mathbb{O}_{u,v}$ is a null matrix of dimension $u \times v$. Matrix inequalities are considered in the sense of Löewner *i.e.* "$\prec 0$" ("$\preceq$") means negative definite (resp. semi-definite) and "$\succ 0$" ("$\succeq$") positive definite (resp. semi-definite). SPD stands for Symmetric Positive Definite. $r_{\mathbb{C}}$ corresponds to the complex stability radius and $r_{\mathbb{R}}$ to the real stability radius.

2 Problem statement

In this part, we will first give the basic tools that are necessary to the comprehension of our contribution. In the second time, we will introduce the modeling of the state matrix subjected to a $\mathcal{JL}$-structured uncertainty and finally we will specify our objective.

2.1 $\mathcal{EMI}$-regions and $\mathcal{D}_{\mathbf{R}}$-stability

In this paragraph, as in the sequel, we will transpose the $\mathcal{D}_R$-stability problem in the complex plane. Thus, each one knows that a continuous system is stable if all the eigenvalues of its state matrix A belong to the half complex plane. Some necessary and sufficient condition for matrix stability results from the second method of Lyapunov applied to the linear systems [4] and is given by the next theorem :

Theorem 1 *Let $A \in \mathbb{R}^{n\times n}$ be the state matrix of a system. The system is asymptotically stable if there exists a SDP matrix $P \in \mathbb{R}^{n\times n}$ such as the following $\mathcal{LMI}$ condition is checked :*

$$A'P + PA \prec 0 \tag{1}$$

However, this stability condition does not take into account the various performances which can be required of the system. Indeed, some specifications could introduce some restrictions, related to the transient response, such as for example the settling time or the damping ratio. For this reason, it is interesting to introduce regions of the complex plane, such as $\mathcal{EMI}$-regions defined as follows [6,5].

Definition 1 *Let a matrix R defined by:*

$$\left\{\begin{array}{l} R = \begin{bmatrix} R_{00} & R_{10} \\ R'_{10} & R_{11} \end{bmatrix} \in \mathbb{R}^{2d\times 2d} \\ R_{11} \succeq 0 \in \mathbb{R}^{d\times d} \end{array}\right. \tag{2}$$

The $\mathcal{D}_R$-region described by :

$$\mathcal{D}_R = \{z \in \mathbb{C} \mid R_{00} + R_{10}z + R'_{10}\tilde{z} + R_{11}z\tilde{z} \prec 0\} \tag{3}$$

is called $\mathcal{EMI}$-region.

d corresponds to the degree of the region and the matrix R_{11} has to be positive definite for technical reasons.
From (1), one can define the $\mathcal{D}_R$-stability of a matrix A as the clustering of its eigenvalues inside the open region $\mathcal{D}_R$. This concept generalizes the one of asymptotic stability and one can extend Th. 1 [6]:

Theorem 2 *Let an $\mathcal{EMI}$-region of degree d described by Def. 1. A matrix $A \in \mathbb{R}^{n\times n}$ is $\mathcal{D}_R$-stable if and only if there is a SDP matrix $P \in \mathbb{R}^{n\times n}$ (resp. as well as a couple of matrices $\{F; G\} \in \{\mathbb{R}^{dn\times dn}\}^2$) such as the following $\mathcal{LMI}$ is checked :*

$$R_{00} \otimes P + R_{10} \otimes (PA) + R'_{10} \otimes (A'P) + R_{11} \otimes (A'PA) \prec 0 \tag{4}$$

or

(resp.
$$\begin{bmatrix} R_{00} \otimes P + (\mathbb{I}_d \otimes A')F' + F(\mathbb{I}_d \otimes A) & R_{10} \otimes P + (\mathbb{I}_d \otimes A')G' - F \\ R'_{10} \otimes P + G(\mathbb{I}_d \otimes A) - F' & R_{11} \otimes P - G - G' \end{bmatrix} \prec 0 \quad) \tag{5}$$

Of course, condition (4) and (5) are equivalent and if the fact of introducing the matrices F and G complicates the computational aspect, it is interesting from a robustness point of view, since it allows, inter alia, to use Parameter-Dependent Lyapunov Functions (PDLF) [6].

Remark 3 *It is interesting to note that the introduction of the $\mathcal{EMI}$-regions make possible to test the discret systems stability by defining $\mathcal{D}_R$ as a unit disc centered on the origin.*

2.2 $\mathcal{JL}$-structured uncertainty modeling

In the previous part, we were interested in the $\mathcal{D}_R$-stability of a nominal matrix A. However, one wants to consider the case where the matrix is subject to an uncertainty. Let $\mathbb{A} = A + \mathbb{E}$ be the uncertain matrix where $\mathbb{A} \in \mathbb{R}^{n\times n}$ is composed of a nominal part A and an additive uncertainty $\mathbb{E} \in \mathbb{R}^{n\times n}$. Nevertheless, to avoid pessimism in the structure of $\mathbb{E}$, it is taken in the form $\mathbb{E} = JEL$ where the uncertain part of $\mathbb{E}$ breaks up into a product of three matrices. $J \in \mathbb{R}^{n\times q}$ and $L \in \mathbb{R}^{r\times n}$ are known matrices fixing the structure of $\mathbb{E}$, and $E \in \mathbb{R}^{q\times r}$ is an unknown matrix. The uncertain matrix is then written :

$$\mathbb{A} = A + JEL \tag{6}$$

This modeling enables us to better specify the uncertainty to which $\mathbb{A}$ is subject to, by conferring some structure to it. If one limits $||E||_2$ by fixing an arbitrarily ρ, this uncertainty is known as "norm-bounded uncertainty" [7].

2.3 Objective of our robust analysis

The objective of our robust analysis is to compute a robustness bound ρ on the norm of $E \in \mathbb{C}^{q\times r}$, largest as possible, which guarantees the $\mathcal{D}_R$-stability of $\mathbb{A}$. In other words, one seeks to maximize the volume of the ball defined by $||E||_2 \prec \rho$ while guaranteeing $\lambda_i(\mathbb{A}) \in \mathcal{D}_R \ \forall\, i \in \{1,..,n\}$.
This optimization of $\mathcal{D}_R$-stability bound will be obtained by solving an $\mathcal{LMI}$ system owing to software $MATLAB^{6.1}$. The technique for the computation of the robustness bound is the subject of the following part.

3 New $\mathcal{D}_\mathbf{R}$-stability bound

In this part, we propose a technique to compute a robustness bound in the case of a $\mathcal{JL}$-structured uncertainty that can be expressed by the following theorem :

Theorem 4 *Let an uncertain matrix $\mathbb{A} \in \mathbb{R}^{n\times n}$ be defined by (6) and an $\mathcal{EMI}$-region be characterized by a given matrix R as in Def. 1. Assume that the nominal matrix $A \in \mathbb{R}^{n\times n}$ is $\mathcal{D}_R$-stable, then the uncertain matrix $\mathbb{A}$ est $\mathcal{D}_R$-stable if :*

$$||E||_2 \leq \rho = (\gamma^\star)^{-\frac{1}{2}} \tag{7}$$

where $\gamma^\star$ is the optimal value ofγ given by:

$$\min_{P,F,G,\gamma} \quad \gamma \quad \text{under the following } \mathcal{LMI} \text{ constraint:}$$

$$\mathcal{N}(P,F,G,\gamma) =$$

$$\left[\begin{array}{c|c|c}
\begin{array}{c} R_{00}\otimes P+ \\ \mathcal{S}(F(\mathbb{I}_d\otimes A)) \end{array} & \begin{array}{c} R_{10}\otimes P+ \\ (\mathbb{I}_d\otimes A')G'-F \end{array} & \cdots \\ \hline
\begin{array}{c} R_{10}'\otimes P+ \\ G(\mathbb{I}_d\otimes A)-F' \end{array} & R_{11}\otimes P - G - G' & \cdots \\ \hline
(\mathbb{I}_d\otimes J')F' & (\mathbb{I}_d\otimes J')G' & \cdots \\ \hline
(\mathbb{I}_d\otimes L) & \mathbb{O}_{dr,dn} & \cdots
\end{array}\right.$$

$$\left.\begin{array}{c|c}
F(\mathbb{I}_d\otimes J) & (\mathbb{I}_d\otimes L') \\ \hline
G(\mathbb{I}_d\otimes J) & \mathbb{O}_{dn,dr} \\ \hline
(-\mathbb{I}_{dq}) & \mathbb{O}_{dq,dr} \\ \hline
\mathbb{O}_{dr,dq} & -\gamma\mathbb{I}_{dr}
\end{array}\right] \prec 0 \tag{8}$$

where $P \in \mathbb{R}^{n\times n}$ is SDP and $\{F;G\} \in \{\mathbb{R}^{dn\times dn}\}^2$.

Proof: Let a matrix S defined by :

$$S = \begin{bmatrix} \mathbb{I}_{dn} & \mathbb{I}_d\otimes\mathbb{A}' & \mathbb{I}_d\otimes L'E' & \mathbb{O}_{dn,dr} \\ \mathbb{O}_{dr,dn} & \mathbb{O}_{dr,dn} & \mathbb{O}_{dr,dq} & \mathbb{I}_{dr} \end{bmatrix} \tag{9}$$

Pre-multiply and post-multiply (8) by, respectively, S et S', it comes:

$$S\mathcal{N}(P,F,G,\gamma)S' \prec 0 \tag{10}$$

We obtain after some simplifications :

$$\left[\begin{array}{l|l} R_{00} \otimes P+ & \\ \mathcal{S}((R_{10} \otimes P)(\mathbb{I}_d \otimes \mathbb{A}))+ & \\ (\mathbb{I}_d \otimes \mathbb{A}')(R_{11} \otimes P)(\mathbb{I}_d \otimes \mathbb{A}) & \\ -(\mathbb{I}_d \otimes L'E')(\mathbb{I}_d \otimes EL) & (\mathbb{I}_d \otimes L') \\ \hline (\mathbb{I}_d \otimes L) & -\gamma \mathbb{I}_{dr} \end{array}\right] \prec 0 \tag{11}$$

By applying Schur's lemma, this is equivalent to :

$$\begin{array}{l} R_{00} \otimes P + \mathcal{S}(R_{10} \otimes (\mathbb{A}P)) + R_{11} \otimes (\mathbb{A}'P\mathbb{A}) \prec \\ \mathbb{I}_d \otimes (L'E'EL - \gamma^{-1}L'L) \end{array} \tag{12}$$

At this stage, if $||E||_2 \prec \rho$ then $L'E'EL - \gamma^{-1}L'L \prec 0$ so:

$$R_{00} \otimes P + \mathcal{S}(R_{10} \otimes (\mathbb{A}P)) + R_{11} \otimes (\mathbb{A}'P\mathbb{A}) \prec 0 \tag{13}$$

This is the $\mathcal{D}_R$-stability condition of the matrix $\mathbb{A}$. $\square$
Minimize γ under the $\mathcal{LMI}$ constraint (8) is a convex problem that is easily solved from a numerical point of view.

Remark 5 *Note that this new condition never uses triangular inequality, what reduces conservatism of computation compared to usual techniques [9,1,2].*

Remark 6 *As we announced in the previous part, matrices F and G have the advantage to take PDLF into account [6]. This technique could thus be easily exploited in the case of a both polytopic and $\mathcal{JL}$-structured uncertainty. We insist on the advantage of introducing F and G for future investigations.*

4 Numerical illustration

In this part, we propose four examples which illustrate on the one hand, the possibilities offered by the uncertainty modelling in the analysis technique, and on the other hand, the weak conservatism induced by the $\mathcal{D}_R$-stability bound computation method. Computations are carried out using a Pentium 900Mhz.

4.1 Example 1

This example results from [8] where it is proposed to compute a stability radius, real or complex, with respect to a region of the complex plane. This example enables us to compare the results resulting from two different analysis techniques : the μ-analysis and the analysis presented in this article.
Let us consider the system modelled by the following matrices :

$$A = \begin{bmatrix} 79 & 20 & -30 & -20 \\ -41 & -12 & 17 & 13 \\ 167 & 40 & -60 & -38 \\ 33.5 & 9 & -14.5 & -11 \end{bmatrix}$$

$$J = \begin{bmatrix} 0.2190 & 0.9347 \\ 0.0470 & 0.3835 \\ 0.6789 & 0.5194 \\ 0.6793 & 0.8310 \end{bmatrix}$$

$$L = \begin{bmatrix} 0.0346 & 0.5297 & 0.0077 & 0.0668 \\ 0.0535 & 0.6711 & 0.3834 & 0.4175 \end{bmatrix}$$

In [8], by using "μ-like" techniques, the authors propose the calculation of a stability radius (or rather a lower bound of this radius). This problem can be considered by our analysis tool because our bound is the inverse of an upper bound of complex μ. In this example, considering the perturbation as a complex one, introduced some conservatism, compared with a real μ-technique.
In fact, in [6], the computed bound corresponding to a real uncertainty was $r_{\mathbb{R}} = 0.5141$ and that corresponding to a complex uncertainty was $r_{\mathbb{C}} = 0.3914$. By our technique we found :

$$\rho = 0.3914$$

in only $0.27s$. This bound equals that obtained by μ-complex analysis and although it lower than the real μ bound, it is far from conservative as shown on Fig. 1 where the poles of matrix $\mathbb{A} = A + JEL$ are plotted for many values of E such as $||E||_2 \leq \rho$.

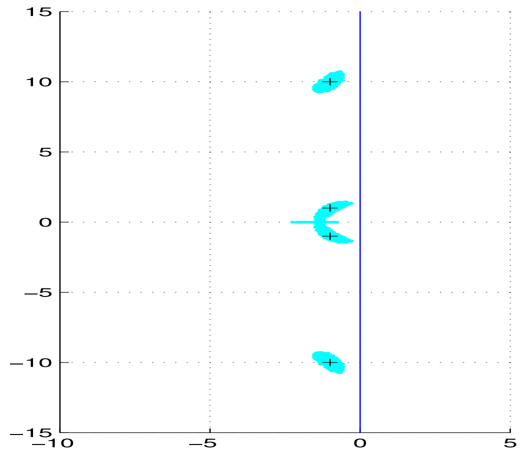

Figure 1: Example 1 with $||E||_2 \leq 0.3914$

4.2 Example 2

This second example also results from [8] and the principle is exactly the same as the one previously presented, put aside that the considered region is not the complex left half-plane any more but the unit disc centered on the origin. In fact, we pass from a stability problem within the Hurwitz meaning to a stability problem within the Schur meaning.
The uncertain matrix is :

$$A = \begin{bmatrix} 4.7527 \times 10^{-1} & 7.5787 \times 10^{-1} & 7.9939 \\ -4.1523 \times 10^{-2} & 8.9051 \times 10^{-1} & 7.5787 \times 10^{-1} \\ -7.5787 \times 10^{-2} & -4.1523 \times 10^{-2} & 4.7527 \times 10^{-1} \end{bmatrix}$$

$$J = \begin{bmatrix} 8.0086 \times 10^{-2} & 4.2994 \times 10^{-2} \\ -1.4704 \times 10^{-3} & 9.4791 \times 10^{-2} \\ -4.2994 \times 10^{-3} & -1.4704 \times 10^{-3} \end{bmatrix}$$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

In [8], the computed bound were $r_{\mathbb{R}} = 1.0374$ and $r_{\mathbb{C}} = 0.74715$. By our approach, we obtain :

$$\rho = 0.74642$$

with a computation time of $0.06s$. One can, once again, note the obtaining of a better robustness bound by using real μ technique. Nevertheless, our approach led, in a short computation time, on a bound which is similar to that resulting from μ-complex. Our bound is, moreover, not very conservative as shown on Fig. 2.

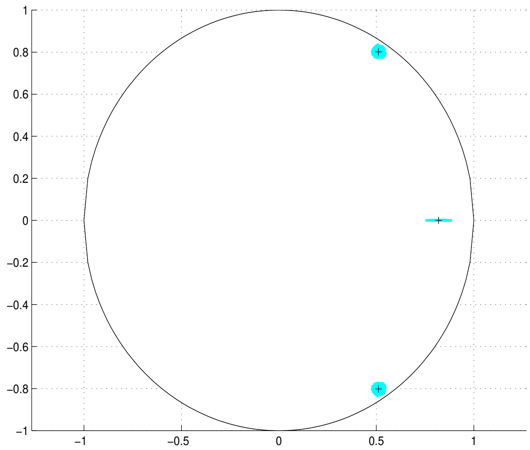

Figure 2: Example 2 with $||E||_2 \leq 0.74642$

4.3 Example 3

This third example treats DC motor with seperate excitation robust analysis. It shows an advantage of the $\mathcal{JL}$-structure. Indeed, let us consider the following modelling obtained using the electric and mechanical equations of the DC motor.

$$A = \begin{bmatrix} -\frac{r}{L} & -\frac{k}{L} \\ \frac{k}{J} & -\frac{f}{J} \end{bmatrix} \quad ; \quad B = \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix};$$

$$C = \begin{bmatrix} 0 & 1 \end{bmatrix}$$

with :

$$\begin{cases} r = 8\,\Omega & : \text{armature resistor} \\ L = 0.3\,H & : \text{armature inductance} \\ k = 0.08 & : \text{motor parameter} \\ M = 0.01\,kg.m^2 & : \text{inertia moment} \\ f = 0.001\,N.m.rad^{-1}.s & : \text{friction coefficient} \end{cases}$$

One applies an output feedback on the model by mean of a proportional corrector $K = -13.2$. The state matrix of the closed loop system can then be written $\mathbb{A} = A + BKC$. Now, let assume that an uncertainty δ_r affects the armature resistor r. We can reveal this uncertainty while writing :

$$\mathbb{A} = A(\delta_r) + BKC$$

$$\Leftrightarrow \mathbb{A} = \begin{bmatrix} -\frac{r+\delta_r}{L} & -\frac{k}{L} \\ \frac{k}{J} & -\frac{f}{J} \end{bmatrix} + BKC$$

$$\Leftrightarrow \mathbb{A} = \begin{bmatrix} -\frac{r}{L} & -\frac{k}{L} \\ \frac{k}{J} & -\frac{f}{J} \end{bmatrix} + BKC + \begin{bmatrix} -\frac{\delta_r}{L} & 0 \\ 0 & 0 \end{bmatrix}$$

From there, one can break up matrix $\mathbb{A}$ into a sum of two matrices : a matrix $A_0 = A + BKC$ representing the nominal part of $\mathbb{A}$ and a matrix $\mathbb{E}$ representing uncertainty such as :

$$\mathbb{A} = A_0 + \mathbb{E} = A_0 + \begin{bmatrix} -\frac{\delta_r}{L} & 0 \\ 0 & 0 \end{bmatrix}$$

It is then easy to write uncertainty in a $\mathcal{JL}$-structured form :

$$\mathbb{A} = A_0 + \begin{bmatrix} -\frac{1}{L} \\ 0 \end{bmatrix} . \delta_r . \begin{bmatrix} 1 & 0 \end{bmatrix}$$

while posing $J = \begin{bmatrix} -\frac{1}{L} \\ 0 \end{bmatrix}$, $L = \begin{bmatrix} 1 & 0 \end{bmatrix}$ *and* $E = \delta_r$.

Our technique thus makes possible to calculate a bound ρ on δ_r allowing to ensure $\mathcal{D}_R$-stability of $\mathbb{A}$. The region considered here is a left open sector, symmetric with respect to the real axis, with its corner at the origin and making an angle of 37° with the imaginary axis so as to guarantee a damping ratio higher than 0.6.
The computed bound is :

$$\rho = 1.2338$$

what corresponds to a very tight result if one refers to Fig. 3. Moreover, the necessary computation time does not exceed $0.22s$.

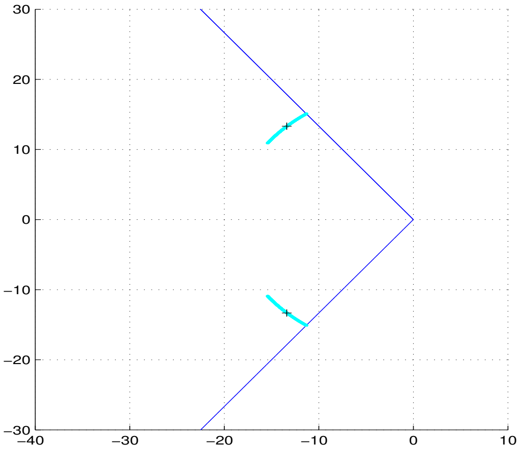

Figure 3: DC motor with $||\delta_r||_2 \leq 1.2338$

4.4 Example 4

In this last example, we are interested in the fragility analysis of a helicopter in “vertical take off landing (VTOL)” modelled as below :

$$A = \begin{bmatrix} -0.0366 & 0.0271 & 0.0188 & -0.4555 \\ 0.0428 & -1.0100 & 0.0024 & -4.0208 \\ 0.1002 & 0.3681 & -0.7070 & 1.4200 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0.4422 & 0.1761 \\ 3.5446 & -7.5922 \\ -5.52 & 4.49 \\ 0 & 0 \end{bmatrix}$$

One assumes that one can measure some components of the state vector, namely :

$$\begin{cases} y_1 = & x_1 & : & \text{horizontal speed} \\ y_2 = & x_2 & : & \text{vertical speed} \\ y_3 = & x_4 & : & \text{inclination angle} \end{cases}$$

We consider an output matrix :

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

In order to place all the closed loop spectrum in the half-plane, a pole placement by output static feedback leads to the following matrix :

$$K = \begin{bmatrix} -12.3506 & -11.3996 & 6.9424 \\ -5.7475 & -4.5881 & 2.6655 \end{bmatrix}$$

The closed loop system state matrix is thus written :

$$\mathbb{A} = A + BKC$$

Being given that we wish to make the closed-loop system fragility analysis, let us reveal an uncertainty on the feedback matrix K such that the closed-loop state matrix becomes :

$$\mathbb{A} = A + B(K + \Delta_K)C$$

$$\Leftrightarrow \mathbb{A} = A + BKC + B\Delta_K C = A_0 + JEL$$

with

$$\begin{cases} A_0 & = & A + BKC \\ J & = & B \\ L & = & C \\ E & = & \Delta_K \end{cases}$$

By applying our analysis technique, we obtain a bound ρ such as $||E||_2 = ||\Delta_K||_2 \leq \rho$ in 0.28s:

$$\rho = 0.3773$$

As we mentionned in the other examples, this result is not very conservative if one judges by the uncertain system poles migration reprented on Fig. 4.

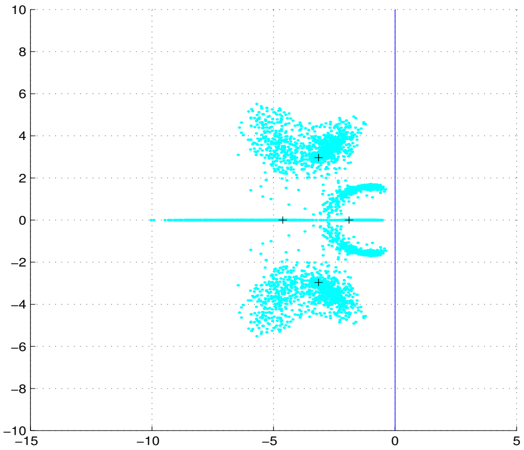

Figure 4: Hlicopter in VTOL with $||\Delta_K||_2 \leq 0.3773$

5 Conclusion

We proposed a technique to compute a robust $\mathcal{D}_R$-stability bound of a matrix subjected to a $\mathcal{JL}$-structured uncertainty. This new computation technique is very tractable from a numerical point of view and the computation time is generally very weak as shown by examples of this article. This last point constitutes some advantage compared with the μ-analysis which requires a much more significant computation time because of the frequency scanning. In addition, the modeling is simple and does not require a particular model transformation. Nevertheless, it should be noted that our approach presents two limits :

- it does not bring any frequential information.
- it does not take into account the reality of E, which leads to an additional conservatism.

$\mathcal{JL}$-structured uncertainty proves to be useful to describe a range of uncertainty sometimes being able to go from the perfectly unknown structure to the "additive-structured" standard formulation. Moreover it allows a fragility analysis induced by a static control law.

The introduction of matrices F and G in the condition makes possible to effectively take into account the structure of $\mathbb{E}$, namely J and L. But these matrices F and G were originally introduced to handle parameter-dependent Lyapunov functions in the case of a polytopic uncertainty, a future work could consist in considering both polytopic and $\mathcal{JL}$-structured uncertainties.