"Bi-polytopic" Lyapunov functions for robust stability and fragility analysis of discrete-time models

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Index Terms

Robust stability analysis, Fragility, Static output feedback, $\mathcal{LMI}$, Parameter-dependent Lyapunov functions.

Abstract

This paper addresses the problem of analysing both robust stability and non fragility induced by a static output feedback control law. The considered models are discrete-time linear state-space representations that are subject to polytopic uncertainties. The only restriction is that the uncertainty affecting the observation matrix is independent on the uncertainty affecting evolution and control matrices. Stability is attested by the existence of a Lyapunov function that is quadratic with respect to the state vector and depends on the uncertain parameters in a "bi-polytopic way". Fragility is also handled in this paper. This works is part of $\mathcal{LMI}$ framework.

1 Introduction

Analysis of uncertain discrete-time linear state-space systems is a major topic in automatic control. The uncertainties can be due to approximations while linearizing the model, to neglected dynamics, or to parameter variation. This last case has been particularly studied in the recent years, especially through the concept of polytopic uncertainty. Only sufficient conditions were proposed in the literature. Indeed, proving stability over a polytope is known to be a $\mathcal{NP}$-hard problem and looking for fewest pessimistic conditions is still a challenge.
One of the most popular concepts is the one of quadratic stability [2] that consists in searching for a single quadratic Lyapunov function available for the whole uncertainty domain. It led to significant but conservative results [16, 3, 9, 10].
More recently, many authors proposed results related to parameter-dependent Lyapunov functions [7, 12, 6]. These functions can depend on parameters in a polytopic way, as in the present work.
It is assumed that the open-loop model is polytopic or, in other words, a convex combination of extreme models. While applying a static output feedback control law, the convexity might not be

preserved if the uncertainty affects both the input and output matrices. Assuming that both input and ouptut uncertainties are independent on each other, the concept of "bi-polytopic stability" will be used to assess robust stability in spite of the non convexity of the closed-loop model. Besides, another additive uncertainty will be considered. It is a parameter-dependent norm-bounded uncertainty that can be introduced to take neglected dynamics into account. Actually, it is mostly introduced to consider a possible unstructured uncertainty on the feedback matrix. Thus, fragility can be analysed. All the proposed conditions are expressed in terms of Linear Matrix Inequalities ($\mathcal{LMI}$).
The paper is organized as follows: the second section is dedicated to the robust stability analysis. The third section considers both robustness and fragility together. In the fourth section, an illustrative example is presented before concluding in the fifth one.

Notations : we denote by M', the transpose of matrix M, by $||M||_2$ its 2-norm induced by the Euclidean vector-norm, *i.e.* the maximal singular value of M. $\mathbf{I}_n$ is the identity matrix of order n, $\mathbf{0}_{u,v}$ is a null matrix of dimension $u \times v$. Matrix inequalities are considered in the sense of Löewner *i.e.* "< 0" means negative definite and "> 0" stands for positive definite.

2 Robust stability analysis

In this section, some significant result relevant to robust stability is recalled [15]. A limit of this results is highlighted. We propose a new condition to overcome this limit in some practical cases.

2.1 Background

Assume that it is aimed to check the stability of the open-loop discrete-time model given by:

$$\left\{ \begin{array}{ccccc} x_{k+1} & = & Ax_k & + & Bu_k \\ y_k & = & Cx_k & & \end{array} \right. \tag{1}$$

where $x(t)$ (or x_k) is a $\mathbb{R}^n$ state vector, $u(t)$ (or u_k) is a $\mathbb{R}^m$ input vector and $y(t)$ (or y_k) is a $\mathbb{R}^p$ output vector. $M = [\,A\ B\,]$ is somewhere in the polytope of matrices $\mathcal{M}$ defined by:

$$\begin{array}{l} \mathcal{M} = \Big\{ \ M \in \mathbb{R}^{n\times(n+m)}\,|\,M \ = \ M(\alpha) \ = \\ \displaystyle\sum_{i=1}^{N_M}(\alpha_i M_i) \ = \ \sum_{i=1}^{N_M}(\alpha_i[\,A_i\ B_i\,]) \ \Big\} \end{array} \tag{2}$$

with α being the vector of barycentric coordinates belonging to set $\Xi(N_M)$ defined by:

$$\Xi(N_M) = \left\{ \ \alpha = \left[\begin{array}{c} \alpha_1 \\ \vdots \\ \alpha_{N_M} \end{array} \right] \in (\mathbb{R}^+)^{N_M} | \sum_{i=1}^{N_M} \alpha_i = 1 \ \right\} \tag{3}$$

In other words, M is some convex combination of N_M known vertices M_i with unknown coordinates α_i.
In the same way, C belongs to a polytope $\mathcal{C}$ defined by:

$$\mathcal{C} = \left\{ \ C \in \mathbb{R}^{p\times n} | C = C(\beta) = \sum_{j=1}^{N_C}(\beta_j C_j),\ \beta \in \Xi(N_C) \ \right\} \tag{4}$$

It is assumed that both formulations are independent. It is first aimed to analyse robust stability of (1).

Theorem 1 *(Particular case of a result of [15]) Let a polytopic uncertain discrete-time model be defined by (1). It is robustly stable if there exist N_M matrices $P_i = P_i' > 0$, $i = 1, ..., N_M$ and a couple of matrices $\{F; G\} \in \{\mathbb{R}^{n\times n}\}^2$ such that, $\forall i \in \{1, ..., N_M\}$:*

$$\begin{bmatrix} -P_i + A_i'F' + FA_i & A_i'G' - F \\ GA_i - F' & P_i - G - G' \end{bmatrix} < 0 \tag{5}$$

Owing to the previous results, stability is attested by the existence of a Lyapunov function of the form:

$$V(x, \alpha) = x'Px \quad \text{with } P = \sum_{i=1}^{N_M} (\alpha_i P_i) \tag{6}$$

This convex matrix P enables to take uncertainty into account and it can be proved following arguments of [12] or [1] that quadratic stability is necessary for condition (5) to be satisfied.

Assume now an output feedback control law is applied:

$$u_k = Ky_k \; \forall k \tag{7}$$

with $K \in \mathbb{R}^{m\times p}$. The closed-loop state matrix is then:

$$A_{cl} = A + BKC \tag{8}$$

Theorem 1 can not be used anymore because A_{cl} also depends on β. Morevover, even if the output uncertainty is included in the same polytopic formulation, the reasonning of the proof can not de developed since linearity with respect to the uncertain parameters is not preserved. The only cases that can be considered are the cases where either B or C is not affected by any uncertainty.

2.2 "Bi-polytopic stability"

In this part, a result is proposed for $N_M \neq 1 \,\&\, N_C \neq 1$.

Theorem 2 *Closed-loop model defined by (1) with (7) is robustly stable if there exist N_M symmetric matrices $\overline{P}_i \in \mathbb{R}^{n\times n}$, $i = 1, ..., N_M$, N_C symmetric matrices $\overline{\overline{P}}_j \in \mathbb{R}^{n\times n}$, $j = 1, ..., N_C$, as well as a couple of matrices $\{F; G\} \in \{\mathbb{R}^{n\times n}\}^2$ such that:*

$$H_{i,j}^{\{d\}} = \left[\begin{array}{c|c} \begin{array}{c} -\overline{P}_i - \overline{\overline{P}}_j + \\ (A_i + B_iKC_j)'F' + \\ F(A_i + B_iKC_j) \end{array} & (A_i + B_iKC_j)'G' - F \\ \hline G(A_i + B_iKC_j) - F' & \overline{P}_i + \overline{\overline{P}}_j - G - G' \end{array}\right] < 0$$
$$\forall \{i; j\} \in \{\{1, ..., N_M\} \times \{1, ..., N_C\}\} \tag{9}$$

and:

$$\tilde{P}_{i,j} = \overline{P}_i + \overline{\overline{P}}_j > 0 \quad \forall \{i; j\} \in \{\{1, ..., N_M\} \times \{1, ..., N_C\}\} \tag{10}$$

Proof: from (9) and (10), it comes, $\forall j \in \{1, ..., N_C\}$:

$$\begin{cases} H_j\{d\} \;=\; \sum_{i=1}^{N_M}(\alpha_i H_{i,j}^{\{d\}}) \;= \\ \left[\begin{array}{c|c} \begin{array}{c} -\overline{P} - \overline{\overline{P}}_j + \\ (A+BKC_j)'F' + \\ F(A+BKC_j) \end{array} & (A+BKC_j)'G' - F \\ \hline G(A+BKC_j) - F' & \overline{P} + \overline{\overline{P}}_j - G - G' \end{array}\right] < 0 \\ \forall j \in \{1, ..., N_C\} \\ \tilde{P}_j \;=\; \sum_{i=1}^{N_M}(\alpha_i \tilde{P}_{i,j}) \;=\; \overline{P} + \overline{\overline{P}}_j \;>\; 0 \end{cases} \tag{11}$$

where $\overline{P} \;=\; \overline{P}(\alpha) \;=\; \sum_{i=1}^{N_M}(\alpha_i \overline{P}_i)$. In the same way, for any instance of $\beta \in \Xi(N_C)$, it comes:

$$\begin{cases} H^{\{d\}} \;=\; \sum_{j=1}^{N_C}(\beta_j H_j^{\{d\}}) \;= \\ \left[\begin{array}{cc} -\overline{P} - \overline{\overline{P}}(A_{cl})'F' + F(A_{cl}) & (A_{cl})'G' - F \\ G(A_{cl}) - F' & \overline{P} + \overline{\overline{P}} - G - G' \end{array}\right] < 0 \\ P \;=\; \sum_{j=1}^{N_C}(\beta_j \tilde{P}_j) \;=\; \overline{P} + \overline{\overline{P}} \;>\; 0 \end{cases} \tag{12}$$

where $\overline{\overline{P}} = \overline{\overline{P}}(\beta) = \sum_{j=1}^{N_C}(\beta_j \overline{\overline{P}}_j)$.

Now premultliply the first inequality of (12) by $[\,\mathbf{I}_n \;\; A_{cl}'\,]$ and postmultiply it by $\left[\begin{array}{c} \mathbf{I}_n \\ A_{cl} \end{array}\right]$. It yields:

$$-P \;+\; A_{cl}'PA_{cl} \;<\; 0 \tag{13}$$

As $P = P' > 0$ (from (12)), $V(x, \alpha, \beta) = x'Px$ is a Lyapunov function for the closed-loop model that is then stable. □

Remark 3 *Lyapunov matrix P is the sum of two convex combinations:*

$$P \;=\; P(\alpha, \beta) \;=\; \sum_{i=1}^{N_M}(\alpha_i \overline{P}_i) \;+\; \sum_{j=1}^{N_C}(\beta_j \overline{\overline{P}}_j) \tag{14}$$

Each term in this sum belongs to a polytope of matrices, one generated by uncertainty on A and B and the other one generated by uncertainty on C. That is why such a stability can be referred to as "bi-polytopic stability". However, it is just a reference to the technique used to derive the Lyapunov function since actually, it is very easy to see with (13) and (14) that $P/2$ is also a Lyapunov function for the closed-loop model and as $\frac{1}{2}(\sum_{i=1}^{N_M}\alpha_i + \sum_{j=1}^{N_C}\beta_j) = 1$, $P/2$ is a convex combination of various $\overline{P}_i$ and $\overline{\overline{P}}_j$.

Remark 4 *Following the same principle, A and C could depend on α and B on β.*

3 Robustness and fragility analysis

In this section, an additive uncertainty is introduced in the model. It is also parameter-dependent. A new $\mathcal{LMI}$ analytical condition is proposed to take it into account. It is then applied to exhibit a non fragility criterion.

3.1 The additive uncertainty

The state matrix of the closed-loop system is now given by:

$$A_{cl} = A + BKC + JEL \tag{15}$$

$M = [\,A\ B\,]$ and C are defined as in section 2. $J \in \mathbb{R}^{n\times q}$ is a parameter-dependent matrix that can be written as follows:

$$J = \sum_{i=1}^{N_M}(\alpha_i\overline{J}_i) + \sum_{j=1}^{N_C}(\beta_j\overline{\overline{J}}_j) \tag{16}$$

where the various α_i and β_j are the coordinates appearing in the expressions of M and C. $L \in \mathbb{R}^{r\times n}$ is defined in the same way:

$$L = \sum_{i=1}^{N_M}(\alpha_i\overline{L}_i) + \sum_{j=1}^{N_C}(\beta_j\overline{\overline{L}}_j) \tag{17}$$

with the same coordinates.
Matrices $\overline{J}_i$, $\overline{\overline{J}}_j$, $\overline{L}_i$ and $\overline{\overline{L}}_j$, $i = 1, ..., N_M$, $j = 1, ..., N_C$ are precisely known. Matrix $E \in \mathbb{R}^{q\times r}$ is an unknwon matrix such that $||E||_2 < \rho$. Such a formulation is a generalisation of the well known norm-bounded uncertainty. It is aimed to test robust closed-loop stability against both polytopic and additive uncertainties while maximizing ρ.

3.2 Robust stability analysis

We state the next theorem:

Theorem 5 *Let a closed-loop system have a state matrix given by (15). It is robustly stable (in the sense of Schur) if:*

$$||E||_2 \leq \rho = (\gamma^\star)^{-\frac{1}{2}} \tag{18}$$

where $\gamma^\star$ is solution to:

$$\min_{\{\gamma,\overline{P}_1,\dots,\overline{P}_{N_M},\overline{\overline{P}}_1,\dots,\overline{\overline{P}}_{N_C},F,G\}} \gamma \tag{19}$$

and where $\gamma \in \mathbb{R}$, $\overline{P}_i = \overline{P}_i' \in \mathbb{R}^{n\times n}$, $i = 1, ..., N_M$, $\overline{\overline{P}}_j = \overline{\overline{P}}_j' \in \mathbb{R}^{n\times n}$, $j = 1, ..., N_C$, $F \in \mathbb{R}^{n\times n}$ and $G \in \mathbb{R}^{n\times n}$ check, for any $\{i;j\}$ in $\{1, ..., N_M\} \times \{1, ..., N_C\}$, the following $\mathcal{LMI}$ system:

$$\left\{\begin{array}{l} \tilde{H}_{i,j}^{\{d\}} = \\ \left[\begin{array}{c|c|c|c} -\overline{P}_i - \overline{\overline{P}}_j + \mathbb{A}_{ij}'F' + F\mathbb{A}_{ij} & \mathbb{A}_{ij}'G' - F & F(\overline{J}_i + \overline{\overline{J}}_j) & \overline{L}_i' + \overline{\overline{L}}_j' \\ \hline G\mathbb{A}_{ij} - F' & \overline{P}_i + \overline{\overline{P}}_j - G - G' & G(\overline{J}_i + \overline{\overline{J}}_j) & \mathbf{0}_{n,r} \\ \hline (\overline{J}_i' + \overline{\overline{J}}_j')F' & (\overline{J}_i' + \overline{\overline{J}}_j')G' & -\mathbf{I}_q & \mathbf{0}_{q,r} \\ \hline \overline{L}_i + \overline{\overline{L}}_j & \mathbf{0}_{r,n} & \mathbf{0}_{r,q} & -\gamma\mathbf{I}_r \end{array}\right] < 0 \\ \tilde{P}_{i,j} = \overline{P}_i + \overline{\overline{P}}_j > 0 \end{array}\right. \tag{20}$$

where $\mathbb{A}_{ij} = A_i + B_iKC_j$.

Proof: consider, $\forall\{\alpha;\beta\} \in \Xi(N_M) \times \Xi(N_C)$, the next combinations involving barycentric coordinates (refering to the proof of theorem 2):

$$\left\{\begin{array}{l} \tilde{H}^{\{d\}} = \sum_{j=1}^{N_C}(\beta_j(\sum_{i=1}^{N_M}(\alpha_i\tilde{H}_{i,j}^{\{d\}}))) = \\ \left[\begin{array}{cccc} -P + (A_{cl})'F' + F(A_{cl}) & (A_{cl})'G' - F & FJ & L' \\ G(A_{cl}) - F' & P - G - G' & GJ & \mathbf{0}_{n,r} \\ J'F' & J'G' & -\mathbf{I}_q & \mathbf{0}_{q,r} \\ L & \mathbf{0}_{r,n} & \mathbf{0}_{r,q} & -\gamma\mathbf{I}_r \end{array}\right] < 0 \\ \\ P = \sum_{j=1}^{N_C}(\beta_j(\sum_{i=1}^{N_M}(\alpha_i\tilde{P}_{i,j}))) = \sum_{i=1}^{N_M}(\alpha_i\overline{P}_i) + \sum_{j=1}^{N_C}(\beta_j\overline{\overline{P}}_j) = P' > 0 \end{array}\right. \tag{21}$$

Premultiplying and postmultiplying $\tilde{H}^{\{d\}}$ respectivley by

$$\left[\begin{array}{cccc} \mathbf{I}_n & A'_{cl} & L'E' & \mathbf{0}_{n,r} \\ \mathbf{0}_{r,n} & \mathbf{0}_{r,n} & \mathbf{0}_{r,q} & \mathbf{I}_r \end{array}\right] \quad \text{and} \quad \left[\begin{array}{cc} \mathbf{I}_n & \mathbf{0}_{n,r} \\ A_{cl} & \mathbf{0}_{n,r} \\ EL & \mathbf{0}_{q,r} \\ \mathbf{0}_{r,n} & \mathbf{I}_r \end{array}\right]$$

does not change the sense of the previous inequality and leads to:

$$\left[\begin{array}{cc} -P + A'_{cl}PA_{cl} - L'E'EL & L' \\ L & -\gamma\mathbf{I}_r \end{array}\right] < 0 \tag{22}$$

which, by virtue of Schur's lemma, is equivalent to:

$$-P + A'_{cl}PA_{cl} - L'E'El + \gamma^{-1}L'E'EL < 0 \tag{23}$$

This inequality holds for $\gamma = \gamma^\star$ and if (18) is satisfied, then:

$$-P + A'_{cl}PA_{cl} < 0 \tag{24}$$

so P is a Lyapunov matrix (of form (14)) ensuring the "bi-polytopic stability" of the closed-loop model. As γ is minimized then ρ is maximized. $\square$

Such a problem is easily solved owing to the various $\mathcal{LMI}$ softwares.

3.3 Application to fragility analysis

Assume that an open-loop system is described by (1) and that the control law (7) is implemented. Assume that because of practical approximations while implementing this law, K is uncertain and is actually of the form $K + E$ where E is unknown. In that case, the closed loop state matrix is:

$$A_{cl} = A + BKC + BEC \tag{25}$$

what reads formulation (15) with:

$$\left\{\begin{array}{l} q = m \\ r = p \\ \overline{J}_i = B_i \quad \text{and} \quad \overline{L}_i = \mathbf{0}_{p,n} \quad \forall i \in \{1,...,N_M\} \\ \overline{\overline{J}}_j = \mathbf{0}_{n,m} \quad \text{and} \quad \overline{\overline{L}}_j = C_j \quad \forall j \in \{1,...,N_C\} \end{array}\right. \tag{26}$$

Hence, theorem 5 can be applied and as ρ is an upper bound for acceptable $||E||_2$, it becomes a non fragility criterion. Both robustness and fragility induced by the control law are analysed together.

4 Illustration

We here illustrate the relevance of our technique on a very simple example which is built from a model borrowed from [9] and modified for our prurpose. The computation was performed with MATLAB on a PC Pentium 266MHz.

A nominal discrete-time open-loop model is given by:

$$\begin{array}{l} A_0 = \begin{bmatrix} -0.50 & 1.00 & 1.00 \\ 1.25 & -0.25 & -1.00 \\ -2.25 & -0.25 & 0.50 \end{bmatrix} ; B_0 = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} \\ C_0 = \begin{bmatrix} 1.0000 & -0.9724 & -0.9592 \end{bmatrix} \end{array} \tag{27}$$

Matrix A_0 is affected by an additive uncertainty and the state matrix is actually $A = A_0 + H_1 \eta H_2$ where $-1 \leq \eta \in \mathbb{R} \leq$ is an unknown parameter and where H_1 and H_2 are given by:

$$H_1 = \begin{bmatrix} 0.3333 \\ 0.1667 \\ 0.1667 \end{bmatrix} \quad ; \quad H_2 = \begin{bmatrix} 0.1944 & 0.0833 & 0.1389 \end{bmatrix} \tag{28}$$

B_0 is also affected by en uncertainty so that $B = B_0 + \eta_1 [0.1\ 0\ 0]'$. Hence, when η describes its range, $M = [A\ B]$ reads polytopic formulation prposed in (2).
The ouput matrix is in fact $C = C_0 + \mu[00.10]$ with $-1 \leq \mu \leq 1$ so that when μ describes its range, C descibes a polutope as defined in (4).
An output feedback gain $K = 1.0421$ is computed so that the closed-loop model is stable. Our aim is to analyse robust stability of the closed-loop system. We are also interested in appreciating the fragility of this gain with respect to stability. According to §3.3, we fix

$$\left\{ \begin{array}{l} \overline{J}_i = B_i \ ; \quad \overline{L}_i = \mathbf{0}_{3,1} \ , \quad i = 1,2 \\ \overline{\overline{J}}_i = \mathbf{0}_{1,3} \ ; \quad \overline{\overline{L}}_j = C_j \ , \quad j = 1,2 \end{array} \right. \tag{29}$$

Matrices B_i and C_j are vertices of the polytopes deduced from the extreme values of η and μ. Using theorem 5, we prove robsut stability and find, in `5.97 s`, an upper bound on the deflection of K:

$$\Delta K \leq \rho = 0.0041 \tag{30}$$

It is not conservative since plotting the poles of uncertain closed-loop models, with $\Delta K \leq \rho$ with taking both plant and feedback uncertainties into account, a pole is located on the unit circle (see figure 1).

This little example shows the efficiency of our technique to analyse both robust stability and non fragility. It particularly highlights the weak conservatism and the little computation time.

5 Conclusion

In this paper, the problem of analysing robust stability of a closed-loop linear discrete-time invariant state-space model was tackled. Provided that the open-loop model is affected by polytopic uncertainties, that uncertainty affecting the ouput matrix is independent on uncertainty affecting input and state matrices, and that a static output feedbak control law is applied to this model, then a sufficient condition for the closed-loop model to be robustly stable with respect to plant and controller uncertainties was proposed. A non fragility criterion was exhibited. The proposed technique is based on

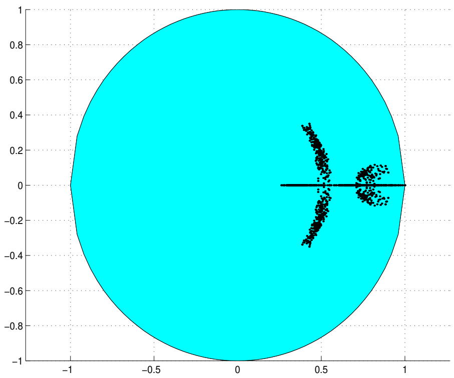

Figure 1: pole migration for $\Delta K \leq \rho$

“bi-polytopic stability” and parameter-dependent norm-bounded uncertainties. It relies on the determination of a parameter-dependent Lyapunov function. It is very tractable from a computational point of view since the condition is formulated as a convex problem that can be solved with classical $\mathcal{LMI}$ tools. A further investigation could be led towards the continuous case using the strong result of [12] and more generally towards closed-loop pole placement analysis with sophisticated clustering regions as in [15, 1].

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