Parameter-dependent Lyapunov robust $\mathcal{D}$-stability bound

Submited to European Control Conference ECC'01

Olivier Bachelier[a], Olivier.Bachelier@univ-poitiers.fr
Denis Arzelier[b], arzelier@laas.fr
Dimitri Peaucelle[b], peaucell@laas.fr

[a]Laii-Esip, Bâtiment de Mécanique, 40 avenue du recteur Pineau, 86022 Poitiers Cedex, France
[b]Laas-Cnrs, 7, av. du Colonel Roche–31077 Toulouse Cedex 4, France

Version 3

Index Terms

LMI, Robustness bound, generalized eigenvalue problem

Abstract

In this paper, the problem of robust matrix root-clustering against additive structured uncertainty is addressed. A bound on the size of the uncertainty domain preserving matrix $\mathcal{D}$-stability is derived from an $\mathcal{LMI}$ approach. A recently proposed sufficient condition for robust matrix $\mathcal{D}$-stability with respect to convex polytopic uncertainty is used. It is relevant to the framework dealing with parameter-dependent Lyapunov functions.

1 Introduction

We aim to analyse the robustness of transient performances of an LTI system that are strongly dependent on the closed-loop state matrix eigenvalues. To ensure some performances on settling time or on damping ratio, the poles are expected to lie in some specified region $\mathcal{D}$ of the complex plane. This property is referred to as $\mathcal{D}$-stability and can be seen as an extension of the concept of asymptotic stability.
Considering an uncertain matrix $\mathbf{A} = A + E$ where E is an additive uncertainty, our purpose is to find the largest bound on the uncertainty domain such that $\mathbf{A}$ is $\mathcal{D}$-stable provided A is already $\mathcal{D}$-stable. Such bounds are called robustness bounds or robust $\mathcal{D}$-stability bounds. We will give a precise characterization of the class of convex regions to be considered. Besides, we will restrict our study to some particular (but very usual) case of structured parametric uncertainty [1]. The research of robustness bounds has started with the first results proposed in [2]. It was later improved in [3, 4, 5, 6] and extended to, for example, Ω-regions ans $\mathcal{LMI}$-regions [7, 8, 9] or even various non-connected regions [10]. All those bounds suffer from some conservatism that is due, for most of them, to two reasons:
• First, those bounds are strongly dependent on the notion of "quadratic stability" [11] *i.e.* robust $\mathcal{D}$-stability is attested by a single matrix for the whole uncertainty domain.
• Second, the expressions of those bounds have been derived from the use of triangular inequality on the norms.
In this paper, we propose to compute a robust $\mathcal{D}$-stability bound in a very different manner. To achieve

our goal, we take benefit of a condition recently introduced in [12] and relying on the use of parameter-dependent Lyapunov functions [13, 14]. This framework helps us to reduce the first source of conservatism. We also formulate our problem as a generalized eigenvalue one. Hence, no expression or robustness bound is required. The use of triangular inequality is avoided what leads to the suppression of the second source of conservatism.

The paper is organized as follows. After this introduction, the second section is dedicated to the problem statement with the presentation of the uncertainty structure as well as our precise purpose, the characterization of the considered regions and an helpful background due to [12]. In the third section, the main result is given: a $\mathcal{D}$-stability bound is computed by solving a generalized eigenvalue problem. Numerical examples are proposed in the fourth section before concluding in the fifth one.

Notations : Throughout the paper, we denote by M^*, the transpose conjugate of M (only tranpose if M is real), by $\otimes$, the Kronecker product and by $\bullet$, the Hadamard product $\mathbf{I}_n$ is the identity matrix of order n and $\mathbf{0}_{u,v}$ (resp. $\mathbf{0}_u$) is a null matrix of dimension $u \times v$ (resp. $u \times u$). $\mathbf{1}_{u,v}$ is a matrix of dimension $u \times v$ all the entries of which equal 1. In the matrix inequalities, "< 0" ("≤ 0") stands for negative (semi negative) definiteness and "> 0" ("≥ 0") stands for positive (semi positive) definiteness.

2 Problem statement

In this section, we introduce the structure of the uncertainty that is taken into account. Then, we state the precise problem we want to tackle and we characterize the set of regions we will consider in the sequel. At last, we recall an helpful condition due to [12] that is a basis for this contribution.

2.1 The uncertainty structure

In this paragraph, a structure for the uncertainty is presented. The first formulation is a simplifed one for the structured parametric uncertainty [1]. We then express it as a particular case of the convex polytopic uncertainty. The uncertain matrix is denoted by:

$$\mathbf{A} = \mathbf{A}(\delta) = (A + E) \in \mathbb{R}^{n \times n} \tag{1}$$

A is the nominal matrix (precisely known) and E is an additive uncertainty structured as follows:

$$E = \sum_{i=1}^{p} (\delta_i E_i) \tag{2}$$

where $\delta = [\delta_1 \cdots \delta_i \cdots \delta_p]^* \in \mathbb{R}^p$ is an unknown parameter vector and where E_i, $i \in \{1, ..., p\}$, are known matrices specifying which matrix entries are affected by the parameter variation. In many contributions, δ is assumed to belong to an hyperrectangular set defined by :

$$\Delta = \{\delta = \begin{bmatrix} \delta_1 \\ \vdots \\ \delta_p \end{bmatrix} \in \mathbb{R}^p | (-\underline{\psi}_i \tilde{\delta} < \delta_i < \bar{\psi}_i \tilde{\delta}, \forall i \in \{1, ..., p\})\} \tag{3}$$

where $\underline{\psi}_i$ and $\bar{\psi}_i$ are positive real specified numbers that define the form of Δ in the $\mathbb{R}^p$-space and where $\tilde{\delta}$ can be seen as the "size" of this hyperrectangle. The nominal value is obtained with $\delta = \mathbf{0}_{p,1}$, *i.e.*:

$$\mathbf{A}(\mathbf{0}_{p,1}) = A \tag{4}$$

It is well known that Δ is in fact a convex hull defined by its vertices $\delta_{[j]}$, $j = 1, ..., N = 2^p$ and that $\mathbf{A}(\delta)$ belongs to a polytope $\mathcal{A}$ defined by:

$$\mathcal{A} = \{\mathbf{A}(\xi) \,|\, \mathbf{A}(\xi) = \sum_{j=1}^{N} (\xi_j A_j); \xi \in \Xi\} \tag{5}$$

with:

$$\Xi = \{\xi = \begin{bmatrix} \xi_1 \\ \vdots \\ \xi_N \end{bmatrix} \in \mathbb{R}^N | (\xi_j \geq 0, \forall j \in \{1, ..., N\}; \sum_{j=1}^{N} \xi_j = 1)\} \tag{6}$$

The extreme matrices of the polytope are then:

$$A_j = \mathbf{A}(\delta_{[j]}) = A + \tilde{\delta}\mathcal{E}_j \quad \forall j \in \{1, ..., N\} \tag{7}$$

where the matrices $\mathcal{E}_j$ can be very easily built owing to the various E_i and to:

$$\underline{\Psi} = \begin{bmatrix} \underline{\psi}_1 \\ \vdots \\ \underline{\psi}_p \end{bmatrix} \in \{\mathbb{R}^+\}^p \quad \& \quad \overline{\Psi} = \begin{bmatrix} \overline{\psi}_1 \\ \vdots \\ \overline{\psi}_p \end{bmatrix} \in \{\mathbb{R}^+\}^p \tag{8}$$

This last polytopic form (5) will be useful for the derivation of our main result.

2.2 Robust matrix root-clustering

In the paper, we are interested in the $\mathcal{D}$-stability of $\mathbf{A}$. We assume that A is already $\mathcal{D}$-stable and are looking for a condition on E that preserves the $\mathcal{D}$-stability of $\mathbf{A}$. As explained in introduction, this condition is a robust $\mathcal{D}$-stability bound *i.e.* a bound on the uncertainty domain. Our problem is to compute $\tilde{\delta}^\star$, the maximum value of $\tilde{\delta}$ preserving the $\mathcal{D}$-stability of $\mathbf{A}$. It is very hard to compute so it may be required to reach a lower bound $\bar{\delta} \leq \tilde{\delta}^\star$. $\bar{\delta}$ is a so-called robust $\mathcal{D}$-stability bound. Our goal is to find the bound $\bar{\delta}^\star \leq \tilde{\delta}^\star$ that is as tight as possible.

2.2.1 Characterization of $\mathcal{D}$-regions and $\mathcal{D}$-stability

This paragraph is dedicated to the characterization of the regions, proposed in [12], that we consider in this paper. We define the matrix R by:

$$R = \begin{bmatrix} R_{00} & R_{10} \\ R_{10}^* & R_{11} \end{bmatrix} \quad \text{with :} \quad \begin{cases} R_{00} = R_{00}^* \in \mathbb{R}^{d \times d} \\ R_{11} = R_{11}^* \geq 0 \in \mathbb{R}^{d \times d} \end{cases} \tag{9}$$

From this matrix, the region $\mathcal{D}$ is defined as the subset of the complex plane described by:

$$\mathcal{D} = \{z \in \mathbb{C} | f_{\mathcal{D}}(z) = R_{00} + R_{10}z + R_{10}^* z^* + R_{11} z z^* < 0\} \tag{10}$$

Remark 1 *: the assumption $R_{11} \geq 0$ is made for technical reasons. Actually, it ensures that $\mathcal{D}$ is a convex region. Then, this formulation is just slightly different from the more classical formulation of $\mathcal{LMI}$-regions [15]. $\mathcal{D}$ can be, for example, vertical shifted half planes, classical and hyperbolic sectors, vertical and horizontal strips, disks and ellipses ...*

An $\mathcal{LMI}$ characterization for $\mathcal{D}$-stability is then:

Theorem 2 *[12]: Let the region $\mathcal{D}$ be defined by (10) with (9). A matrix $A \in \mathbb{R}^{n \times n}$ is $\mathcal{D}$-stable* **if and only if** *there exists a symmetric positive definite matrix $P \in \mathbb{R}^{n \times n}$ such that the following $\mathcal{LMI}$ holds:*

$$\begin{aligned} \mathcal{M}_{\mathcal{D}}(A, P) = R_{00} \otimes P + R_{10} \otimes (PA) + \\ R_{10}^* \otimes (A^* P) + R_{11} \otimes (A^* P A) < 0 \end{aligned} \tag{11}$$

2.2.2 Robust $\mathcal{D}$-stability analysis

The polytope $\mathcal{A}$ is said to be robustly $\mathcal{D}$-stable, $\mathcal{D}$ being defined as in (10) with (9), if and only if $\mathbf{A}(\xi)$ is $\mathcal{D}$-stable for any instance of $\xi \in \Xi$. In the Lyapunov context, such a property is equivalent to:

Lemma 1 *: $\mathcal{A}$ is robustly $\mathcal{D}$-stable* **if and only if***:*

$$\forall \xi \in \Xi, \exists P(\xi) = P^*(\xi) > 0 \in \mathbb{R}^{n\times n} | \mathcal{M}_{\mathcal{D}}(\mathbf{A}(\xi), P(\xi)) < 0 \tag{12}$$

It is well known that checking inequality (12) over the whole polytope is known to be an $\mathcal{NP}$-hard problem. However, in [12], extending an original idea presented in [14], a powerful and tractable sufficient condition is proposed.

Theorem 3 *[12, Theorem 4]: Let the polytope $\mathcal{A}$ and the region $\mathcal{D}$ be respectively defined by (5) with (6) and (10) with (9). If there exist matrices $F \in \mathbb{R}^{nd\times nd}$ and $G \in \mathbb{R}^{nd\times nd}$ as well as N symmetric positive definite matrices $P_j \in \mathbb{R}^{n\times n}$, $j \in \{1, ..., N\}$, such that $\forall j \in \{1, ..., N\}$:*

$$\mathcal{N}_{\mathcal{D}}(A_j, P_j, F, G) \ < \ 0 \tag{13}$$

with:

$$\mathcal{N}_{\mathcal{D}}(A_j, P_j, F, G) \ = \ \begin{bmatrix} R_{00} \otimes P_j + F(\mathbf{I}_d \otimes A_j) + (\mathbf{I}_d \otimes A_j^*)F^* & R_{10} \otimes P_j + (\mathbf{I}_d \otimes A_j^*)G - F \\ R_{10}^* \otimes P_j + G^*(\mathbf{I}_d \otimes A_j) - F^* & R_{11} \otimes P_j - G - G^* \end{bmatrix} \tag{14}$$

then $\mathcal{A}$ is robustly $\mathcal{D}$-stable.

The proof is of course achieved in [12] but we must however explain, for clarity, that the previous condition is sufficient for the existence of a parameter-dependent Lyapunov matrix $P(\xi)$ formulated as a convex combination of the various P_j, $j \in \{1, ..., N\}$:

$$P(\xi) \ = \ \sum_{j=1}^{N} (\xi_j P_j) \quad \forall \xi \in \Xi \tag{15}$$

It is noticeable that, following the reasonning achieved in the proof of theorem 3 [12], it appears that $\mathcal{N}_{\mathcal{D}} < 0$ holds over the whole polytope (for some nominal matrix for instance), *i.e.*:

$$\mathcal{N}_{\mathcal{D}}(\mathbf{A}(\xi), P(\xi), F, G) \ < \ 0 \quad \forall \xi \in \Xi \tag{16}$$

In more classical conditions relevant to "quadratic stability", the robust $\mathcal{D}$-stability is attested by the existence of a single Lyapunov matrix for the whole polytope while in this work, the Lyapunov matrix depends on ξ. It leads to a strong reduction of conservatism. Actually, the flexibility provided by F and G is significant since, following arguments given in [14], it can be proved that quadratic $\mathcal{D}$-stability is necessary for condition (13) to hold while the contrary is of course wrong.

3 A new robust $\mathcal{D}$-stability bound

In this section, the main result of the paper is presented. The search for a tight robust $\mathcal{D}$-stability bound is formulated as a generalized eigenvalue problem. As previously mentionned, we aim to compute $\bar{\delta}^\star$, a lower bound of $\tilde{\delta}^\star$ that is as tight as possible. Actually, $\bar{\delta}^\star$ is the size of the largest Δ such that sufficient condition (13) holds. We assume that A (the nominal matrix) is $\mathcal{D}$-stable. We also assume that $\mathbf{A}(\delta)$ is robustly

$\mathcal{D}$-stable for any δ of Δ corresponding to some value of $\tilde{\delta}$. So, the corresponding polytope $\mathcal{A}$ is robustly $\mathcal{D}$-stable and there exist N symmetric positive definite matrices P_j, $j \in \{1, ..., N\}$ as well as a couple of matrices $\{F; G\} \in \{\mathbb{R}^{nd \times nd}\}^2$ such that:

$$\mathcal{N}_{\mathcal{D}}(A_j, P_j, F, G) < 0 \quad \forall j \in \{1, ..., N\} \tag{17}$$

Taking into account the structure of the uncertainty as defined by equation (7) yields, $\forall j \in \{1, ..., N\}$:

$$\mathcal{N}_{\mathcal{D}}(A + \tilde{\delta}\mathcal{E}_j, P_j, F, G) < 0 \tag{18}$$

We define:

$$\mathcal{P}_j = \frac{P_j - P}{\tilde{\delta}} \Leftrightarrow P_j = \tilde{\delta}\mathcal{P}_j + P \quad \forall j \in \{1, ..., N\} \tag{19}$$

where P is any Lyapunov matrix proving the stability of the nominal matrix A. Indeed, $P \in \mathbb{R}^{n \times n}$ is a symmetric positive definite matrix satisfying:

$$\mathcal{N}_{\mathcal{D}}(A, P, F, G) < 0 \tag{20}$$

It comes, from (18), $\forall j \in \{1, ..., N\}$:

$$\mathcal{N}_{\mathcal{D}}(A, P, F, G) + \tilde{\delta} \begin{bmatrix} R_{00} \otimes \mathcal{P}_j + F(\mathbf{I}_d \otimes \mathcal{E}_j) + (\mathbf{I}_d \otimes \mathcal{E}_j^*)F^* & R_{10} \otimes \mathcal{P}_j + (\mathbf{I}_d \otimes \mathcal{E}_j^*)G \\ R_{10}^* \otimes \mathcal{P}_j + G^*(\mathbf{I}_d \otimes \mathcal{E}_j) & R_{11} \otimes \mathcal{P}_j \end{bmatrix} < 0 \tag{21}$$

what can be written:

$$\mathcal{N}_{\mathcal{D}}(A, P, F, G) + \tilde{\delta}\mathcal{L}_{\mathcal{D}}(\mathcal{E}_j, \mathcal{P}_j, F, G) < 0 \quad \forall j \in \{1, ..., N\} \tag{22}$$

Let λ be defined as $\lambda = \tilde{\delta}^{-1}$. It is easy to see that finding the maximal $\overline{\delta}^\star$ of $\tilde{\delta}$ such that (18) holds is equivalent to find $\overline{\delta}^\star = {\lambda^\star}^{-1}$ with:

$$\lambda^\star = \min_{P, \{\mathcal{P}_j\}, F, G} \lambda \quad \text{under the constraints:}$$

$$\begin{cases} \mathcal{L}_{\mathcal{D}}(\mathcal{E}_j, \mathcal{P}_j, F, G) < -\lambda\mathcal{N}_{\mathcal{D}}(A, P, F, G) & \forall j \in \{1, ..., N\} \\ P_j > 0 \Leftrightarrow -\mathcal{P}_j < \lambda P & \forall j \in \{1, ..., N\} \\ \mathcal{N}_{\mathcal{D}}(A, P, F, G) < 0 & \\ P > 0 & \end{cases} \tag{23}$$

As $\mathcal{N}_{\mathcal{D}}(A, P, F, G) < 0$ and $P > 0$, problem (23) is typically a generalized eigenvalue problem that can be solved, for example, by the function `gevp` of the LMI Toolbox of Matlab.

Remark 4 : *At this stage, we shall notice that all this work can be extended to non-connected regions $\mathcal{D}$ resulting from the union of possibly disjoint convex subregions. This can be achieved following about the same reasonning as the one developped in [16].*

Compared with many other robustness bounds, no expression of $\overline{\delta}^\star$ is given, since it is the result of an optimization problem, whereas for most of those bounds, the bound expression is derived from the application of triangular inequality on the 2-norm of matrices inducing conservatism. Besides, they are obtained using NSC for nominal matrix $\mathcal{D}$-stability relying on quadratic $\mathcal{D}$-stability while in our approach, bounding the domain by $\overline{\delta}^\star$ is necessary and sufficient for the existence of an affine parameter-dependent Lyapunov function of the form (15) ensuring $\mathcal{D}$-stability over the whole domain. Besides, even if the existence of such functions is sufficient for robust $\mathcal{D}$-stability, it is less conservative than quadratic $\mathcal{D}$-stability. At last, the optimization problem is numerically tractable. Considering these reasons, we claim that our bound may turn to be one of the least conservative in the literature, from a statistical point fo view, with a low computational cost.

4 Illustrative examples

In this section, we illustrate the previous analytical results by some simple numerical examples. All the computation is achieved on a PC Pentium 166MHz with MATLAB softwares.

4.1 First example

The second example is borrowed from [17]:

$$\mathbf{A} = \mathbf{A}(\delta) = A + \sum_{i=1}^{2} (\delta_i E_i) \tag{24}$$

with:

$$A = \begin{bmatrix} -2 & 0 & -1 \\ 0 & -3 & 0 \\ -1 & -1 & -4 \end{bmatrix}; \tag{25}$$

$$E_1 = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 1 \end{bmatrix}; E_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix} \tag{26}$$

We first consider the classical problem of computing a robust stability bound with:

$$\underline{\Psi} = \overline{\Psi} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \tag{27}$$

The authors of [17] found $\bar{\delta}^\star = 1.5533$. For this example, the most tractable technique is the one proposed in [5] that gives the good value, namely 1.7499, with a little computation time. However, let us notice that this method is only available for stability bounds while our method is available for other regions and for this example, it leads to the same value in only `18.3s`.

• But for this example, we want to insist on the interest of using the vectors $\underline{\Psi}$ and $\overline{\Psi}$. Assume that the designer expects a greater variation of δ_2, compared to the variation of δ_1. It is more appropriate to consider that δ varies in a hyperrectangle rather than in an hypercube (a square for this example). For instance, assume now that:

$$\underline{\Psi}_2 = \overline{\Psi}_2 = 2 \tag{28}$$

For this new instance, we get $\bar{\delta}^\star = 1.5$. It is of course less than 1.7499 but let us now compare the size of the corresponding square and rectangle in the parameter space. This comparison is graphically expressed on figure 1.

• A step further can be made owing to those specified vectors $\underline{\Psi}$ and $\overline{\Psi}$. Not only they easily enable to specify symmetric hyperrectangles (what is often less easy in other techniques) but they also allow to consider nonsymmetric hyperrectangles (what is very hard, even impossible in other techniques dealing with affine parametric uncertainty). For instance, assume that the designer rather thinks that δ_1 and δ_2 are more likely to be negative. He can specify, for instance:

$$\underline{\Psi} = \begin{bmatrix} 10 \\ 10 \end{bmatrix} \quad ; \quad \overline{\Psi} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \tag{29}$$

The computed value of $\overline{\delta}^{\star}$ is now 1.4052.

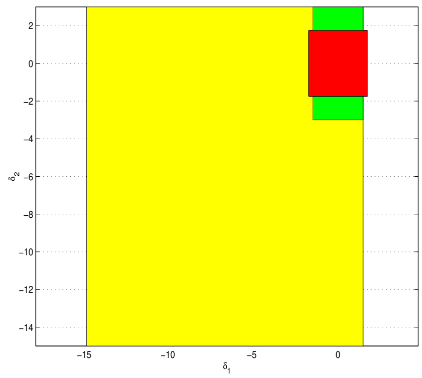

Figure 1: Centred square, centred rectangle and nonsymmetric rectangle in the parameter-space.

This time again, the comparison of the size of the various domains is significant on figure 1 where it is easy to see that $\underline{\Psi}$ and $\overline{\Psi}$ can be very efficiently specified for robust analysis.

4.2 Second example

This example consists in analysing the robustness induced by a control law. It is extracted from [18, page 128]. The open-loop system is described by:

$$\begin{array}{c} \dot{x}(t) \;=\; \mathbf{A}_0 x(t) + \mathbf{B}u(t) \;= \\ (A_0 + \sum_{i=1}^{2}(\delta_i E_i))x(t) + (B + \delta_3 B_3)u(t) \end{array} \tag{30}$$

where:

$$A_0 = \begin{bmatrix} -0.0366 & 0.0271 & 0.0188 & -0.4555 \\ 0.0482 & -1.0100 & 0.0024 & -4.0208 \\ 0.1002 & 0.2855 & -0.7070 & 1.3229 \\ 0.0000 & 0.0000 & 1.0000 & 0.0000 \end{bmatrix}; \tag{31}$$

$$B = \begin{bmatrix} 0.4422 & 0.1761 \\ 3.0477 & -7.5922 \\ -5.5200 & 4.4900 \\ 0.0000 & 0.0000 \end{bmatrix}; \tag{32}$$

$$A_1 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0.1603 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}; \tag{33}$$

$$A_2 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.5820 \\ 0 & 0 & 0 & 0 \end{bmatrix}; \tag{34}$$

$$B_3 \;=\; \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}^* \tag{35}$$

In [18, page133], the following state static feedback control law is computed:

$$K \;=\; \begin{bmatrix} 0.3619 & 0.5234 & 0.9017 & 0.4816 \\ 6.3522 & 1.0309 & -0.0276 & -5.0517 \end{bmatrix} \tag{36}$$

Hence, the closed-loop uncertainty structure depends on the entries of K. Let us denote by $\mathbf{A}$, the uncertain closed-loop state matrix. We have:

$$\mathbf{A} \;=\; A + E \tag{37}$$

with:

$$A \;=\; A_0 + BK\,;\, E \;=\; \sum_{i=1}^{3}(\delta_i Ei)\,;\, E_1 \;=\; A_1\,;\, E_2 \;=\; A_2$$

$$E_3 \;=\; \begin{bmatrix} 0 & 0 & 0 & 0 \\ k_{1,1} & k_{1,2} & k_{1,3} & k_{1,4} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{38}$$

(the scalar $k_{i,j}$ denotes the entry of K that is located at the i^{th} row and the j^{th} column).

The considered region is the disk of centre $(-6;0)$ and of radius 5.5. It can be checked that the eigenvalues of A lie in that disk. In [18], it is proved that those of $\mathbf{A}$ also lie in the disk for $|\delta_i| < 1.9510, \forall i \in \{1,...,3\}$. Assuming that

$$\underline{\Psi} \;=\; \overline{\Psi} \;=\; \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \tag{39}$$

and applying the method presented in this paper leads to a better value:

$$\bar{\delta}^\star \;=\; 2.5752 \tag{40}$$

(`Computation time: 12min07s`)

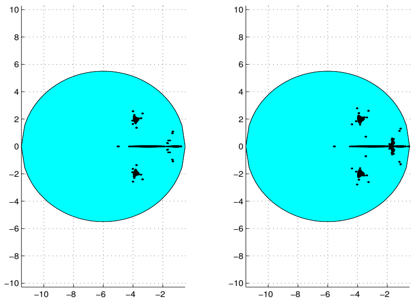

Figure 2: Poles of $\mathbf{A}(\delta)$ with $\delta \leq 1.9510$ and $\delta \leq 2.5752$

The improvement brought by this new method compared with the result of [18, 10] is highlighted by figure 2 where the pole migration with respect to delta is plotted. It can be seen that with the presented method, the found bound nearly correponds to the necessary and sufficient condition since with the second root-locus, some points reach the boundary of $\mathcal{D}$.

5 Conclusion

In this paper, we have proposed a new method to compute robust $\mathcal{D}$-stability bounds in the case of structured parametric uncertainties. No expression of the bound is given. Its value is just the solution of a generalized eigenvalue problem that can be solved with the various $\mathcal{LMI}$ tools. Although this method may not be tractable for a high number of uncertain parameters, numerical illustrations highlighted that it leads to less conservative results than lots of other techniques. This improvement is due to the implicit use of parameter-dependent Lyapunov functions whereas previous bounds were often obtained via a quadratic approach where only a single Lyapunov function attests the $\mathcal{D}$-stability over the whole uncertainty domain.

References

[1] A. Tesi and A. Vicino, "Robust stability of state-space models with structured uncertainty," *IEEE Transactions on Automatic Control*, vol. 35, no. 2, pp. 191–195, February 1990.

[2] R. V. Patel and M. Toda, "Quantitative measures of robustness for multivariable systems," in *Joint Automatic Control Conference*, TP-8A, San Francisco, CA, 1980.

[3] M. G. Safonov, "Stability margins for diagonally perturbed multivariable feedback systems," *IEE, Proc., Pt. D.*, vol. 129, pp. 251–256, November 1982.

[4] R. K. Yedavalli, "Improved mesures of stability robustness for linear state space models," *IEEE Transactions on Automatic Control*, vol. 30, no. 6, pp. 577–579, June 1985.

[5] ——, "Flight control application of new stability robustness bounds for linear uncertain systems," *Journal of Guidance Control and Dynamics*, vol. 16, no. 6, pp. 1032–1037, November-December 1993.

[6] J. D. Gardiner, "Computation of stability bounds for state-space models with structured uncertainty," *IEEE Transactions on Automatic Control*, vol. 42, no. 2, pp. 253–256, February 1997.

[7] R. K. Yedavalli, "Robust root clustering for linear uncertain systems using generalized Lyapunov theory," *Automatica*, vol. 29, no. 1, pp. 237–240, January 1993.

[8] W. Bakker, J. S. Luo, and A. Johnson, "Performance robustness bounds for linear systems to guarantee root-clustering in second order subregions of the complex plane," in *32nd CDC*, San Antonio Texas USA, December 1993, pp. 3467–3472.

[9] O. Bachelier and B. Pradin, "Robust pole placement in a specified convex region of the complex plane," in *IFAC Conference SSC'98*, Nantes, France, July 1998.

[10] ——, "Bounds for uncertain matrix root-clustering in a union of subregions," *International Journal of Robust and Nonlinear Control*, vol. 9, no. 6, pp. 333–359, May 1999.

[11] B. R. Barmish, "Necessary and sufficient conditions for quadratic stabilizability of an uncertain system," *Journal of Optimization Theory and Applications*, vol. 46, no. 4, pp. 399–408, August 1985.

[12] D. Peaucelle, D. Arzelier, O. Bachelier, and J. Bernussou, "A new robust D-stability condition for real convex polytopic uncertainty," *Systems & Control Letters*, vol. 40, no. 1, pp. 21–30, May 2000.

[13] E. Féron, P. Apkarian, and P. Gahinet, "Analysis and synthesis of robust control systems via parameter dependant Lyapunov functions," *IEEE Transactions on Automatic Control*, vol. 41, pp. 436–442, 1996.

[14] J. C. Geromel, M. C. de Oliveira, and L. Hsu, "LMI characterization of structural and robust stability," *Linear Algebra and its Applications*, vol. 285, pp. 69–80, 1998.

[15] M. Chilali and P. Gahinet, "H_∞ Design with Pole Placement Constraints : An LMI Approach," *IEEE Transactions on Automatic Control*, vol. 41, no. 3, pp. 358–367, March 1996.

[16] O. Bachelier, D. Peaucelle, D. Arzelier, and J. Bernussou, "A precise robust matrix root-clustering analysis with respect to polytopic uncertainty," in *Proceedings of ACC*, Chicago - USA, July 2000.

[17] K. Zhou and P. P. Khargonekar, "Stability robustness bounds for linear state-space models with structured uncertainty," *IEEE Transactions on Automatic Control*, vol. 32, no. 7, pp. 621–623, July 1987.

[18] O. Bachelier, "Commande des systèmes linéaires incertains : placement de pôles robuste en D-stabilité," INSA, Toulouse-France, Thse de doctorat, 1998.