New trends on multidimensional systems and their applications in control theory and signal processing*

– Proposal for a CIRM Mini-Workshop (small group) –

Organizing Committee:

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Abstract

In 2014, the French National Research Agency (ANR) awarded the four year MSDOS project that investigates the stability and stabilization of multidimensional systems. This is the first major work on multidimensional systems funded in France on a difficult interdisciplinary project that requires expertise in information sciences (control systems, signal processing), pure mathematics (algebraic geometry, analysis, partial differential equations) and computer science (computer algebra). One of the main purposes of this workshop is therefore to bring closer the mathematician and control communities and focus not only on the theoretical aspects of multidimensional systems but also in their applications both in control theory and signal processing. Four different aspects of the work will be highlighted: two mainly theoretical - Lyapunov theory for linear and nonlinear multidimensional systems and links with partial differential equations, advances on structural stability and stabilization problems based on a constructive version of Deligne's theorem - and two mainly practical - applications to repetitive systems, applications to spatially distributed systems.

Preferred period:	06-10 June 2016
Alternative period:	25-29 April 2016
or:	15-19 February 2016

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1 Aims of the workshop

The idea of analyzing *multidimensional systems* (also called *nD systems*) was born in an environment where both engineers and the researchers shared the same interests. The problem of engineers facing more and more complex models was solved by researchers who understood that these systems could be modeled and controlled using a multidimensional approach.

*Nouvelles avancées dans le domaine des systèmes multidimensionnels et de leurs applications en théorie du contrôle et traitement du signal.

Indeed, most systems have a natural tendency to have multiple dimensions which are often neglected for simplicity. So instead of using 1D digital filter [8], scientists started to look for the implications of digital filters with 2 or more independent variables [8]. A *multidimensional system* was then defined as a system in which information propagates in a n-dimensional space rather than in a single independent direction (usually the time axis for standard 1D systems) [9, 11, 10]. Mathematically, they can be described by different types of continuous or discrete functional systems: systems of partial differential equations, systems of multivariate difference equations, Fornasini-Marchesini models, Roesser models, transfer matrices with multivariate rational functions, multidimensional filters, multidimensional signals (e.g. images), spatially interconnected systems, ... The class of systems that can be modeled as multidimensional systems is therefore very large, see [9, 20, 46, 11, 10] and the references therein.

Nowadays, *multidimensional systems theory* is a common branch of control theory and signal processing which provides researchers with a source of problems having a rich interplay with pure mathematics (e.g., complex analysis in $\mathbb{C}^n$, algebraic geometry, analysis of partial differential equations), computer algebra and scientific computing. To state only one important connection between multidimensional systems theory and computer algebra, we can note that the first complete account of the so-called *Gröbner bases*, nowadays a cornerstone of the symbolic computation − which are implemented in most of the computer algebra systems (e.g., `Maple`, `Mathematica`, `Singular`, `Magma`, `Macaulay2`, `Cocoa`) − was first published by B. Buchberger [12] in the book dedicated on multidimensional systems and was edited by N. K. Bose [11].

In France, this research area has not been part of any workgroup yet (e.g., GdR MACS), but it has been studied by some independent French scientists that organized the first IEEE workshop on multidimensional systems in France (Poitiers, September 2011). This conference contributed to the genesis of the ANR proposal MSDOS (*Multidimensional Systems: Digression On Stability*), SIMI 3, which was awarded in 2013 for the period 2014-2017 (`http://www.lias-lab.fr/perso/nimayeganefar/doku.php?id=welcome`). For this ANR, a precise state of art on multidimensional systems theory was made and open problems on stabilities and stabilization problems were raised. Using recent progress in different mathematical directions (Lyapunov theory, algebraic geometry, analytic geometry, symbolic computation, constructive module theory), in collaboration with international groups, some of these problems can now be tackled.

The goal of the mini-workshop proposal is to gather international specialists (Australia, France, Germany, Poland, USA, ...) of multidimensional systems, control theory, signal processing and computational algebraic geometry, PhD and Postdoc students, ..., to discuss and exchange about the recent achievements in this field. An important issue of the ANR MSDOS is to attract the attention of the French community by alluring new colleagues into the field. Getting a mini-workshop at Luminy (France) on this particular field will be a great opportunity for promoting multidimensional systems theory in France. We have great expectations that discussions developed in the very pleasant place of the CIRM will produce fruitful results and that new collaborations will arise.

2 Tutorials and talks

2.1 Tutorial talks

The mini-workshop will be organized around 5 tutorials which aim at presenting the state of art on important problems and applications in multidimensional systems theory to the general audience (which includes PhD and Postdoc students). Questions on the stability and stabilization of multidimensional systems will be emphasized. The tutorials will be given by international specialists of the fields. In particular, the organizers will invite them to join their expertise to give common lectures and write joined lecture notes. These notes will be the basis of a proposal for a book which will take the form of a Lecture Notes in Control and Information Sciences (LNCIS) and it will be proposed to Springer. This book will aim at disseminating the tutorials and new results not only to the nD community but also to the communities of systems theory, control theory, signal processing, mathematics and computer algebraists.

Here is a preliminary list of tutorials.

1. **Stability and stabilization problems of multidimensional systems**, Eva Zerz (Monday, 2 hours)

2. **Lyapunov techniques for linear and nonlinear multidimensional systems**, Emmanuel Moulay, Nader Yeganefar, Nima Yeganefar (Tuesday, 2 hours)

3. **Identification in multidimensional systems, overview and challenges**, José Ramos (Wednesday, 2 hours).

4. **Applications: Spatially distributed systems and differential time-delay systems**, Alban Quadrat, Hugues Mounier (Thursday, 2 hours)

5. **Applications: Repetitive processes**, Wojciech Paszke (Friday, 2 hours)

The tutorials are planned for the morning sessions.

2.2 Talks

In the afternoon sessions, two to three short talks (from 45 minutes to 60 minutes) will be given on more specific topics and will present important recent results, on-going works and open questions. The organizers will pay a particular attention to leave time for questions and discussions in between talks.

Here is a preliminary list of the talks.

1. Olivier Bachelier, "On the necessary and sufficient condition for stability and stabilization of nD systems using an S-procedure approach".

2. Messaoud Benidir, "Stability of multidimensional filters and signal processing".

3. Yacine Bouzidi, "An overview of computational algebraic geometry techniques for the study of structural stability".

4. Thomas Cluzeau, "Equivalences and solvability of different models of nD systems".

5. Didier Henrion, "Polynomial optimal control and semidefinite optimization".

6. Alban Quadrat, "Towards a constructive version of Deligne's theorem and its applications to multidimensional systems theory".

7. Fabrice Rouillier, "Computational algebraic geometry and the check for the structural stability".

8. Francisco Silva, "Link between partial differential equations and multidimensional systems".

Wednesday afternoon will be free so that the participants have time to collaborate with one another and have a walk in the beautiful Calanques de Cassis.

3 Preliminary scientific program

The main scientific objectives of the mini-workshop are to draw a state of art on the following main problems concerning multidimensional systems theory.

3.1 Lyapunov theory of linear and nonlinear multidimensional systems

Lyapunov theory is a powerful set of tools that has been used for almost a century by the control community. The main idea behind the work of Lyapunov is to know how the solutions will behave without explicitly computing them. An energy-like function called Lyapunov function needs to verify certain criteria (e.g., positive definite and its derivative negative definite) in order to conclude on the stability of the considered equilibrium point. Several extensions of this theory have been proposed in the nD community but so far a complete work on the subject has been missing especially in the nonlinear case. The difficulty lies in the intricate mathematical background (algebra and analysis) required in order to extend the existing theorems to the nD case. Open questions remain: how do we define Lyapunov stability, asymptotic stability, exponential stability, practical stability, uniform stability, etc.? Which theorems based on Lyapunov theory can we give in order to check/guarantee these stability conditions? Ultimately we would like to build a complete stability theory similar to what is found in the 1D case [22]. There have been progress in recent works on this difficult problems (see for instance [43, 44, 23, 24, 40]), which will be summarized during the tutorials.

3.2 Constructive version of Deligne's theorem & structural stability and stabilization problems

A main issue in stabilization problems of multidimensional systems is to develop a constructive version of one of Pierre Deligne's results (reproduced in [21]; see also [13]) which, when combined with algebraic results developed in [33, 34], proves that every structural stabilizable system (i.e., system admitting a controller such that the closed-loop transfer matrix has no poles in the complex unit poly-disc $\mathbb{D}^n$ of $\mathbb{C}^n$ [19]) admits doubly coprime factorizations. This result was conjectured by Z. Lin [29, 30] who solved this problem for particular classes of nD systems. The proof in [21] does not give the mean to compute the corresponding doubly coprime factorizations. To achieve this goal, we first need to constructively compute in the ring A of rational functions without poles in the complex unit poly-disc $\mathbb{D}^n$. To do that, we need to investigate how to combine Gröbner basis techniques [7, 12] with techniques coming from computational algebraic geometry [1, 2, 6, 15, 28] such as Cylindrical Algebraic Decomposition (CAD). Then, the module structure of the ring A has to be effectively investigated into since Deligne's result asserts that finitely generated projective modules over A are free. In particular, we have to investigate how bases of free A-modules can be explicitly computed: a basis gives a doubly coprime factorization (the projectivity property being equivalent to structural stabilizability) [33, 34]. These results allow one to constructively extend classical methods developed for 1D systems [41, 45] to nD systems (e.g., computation of the Youla-Kučera parametrization of all stabilizing controllers; use of this parametrization to transform nonlinear optimal problems (e.g., H^2, H^∞) into affine and thus convex optimal problems, ...). For a survey of these methods for nD systems, based on the knowledge of doubly coprime factorizations, see [3]. These different issues will be studied during the mini-workshop.

3.3 Applications to repetitive systems

The last decade, Repetitive Processes (RP) and Iterative Learning Control (ILC) have been successfully analyzed using the nD framework and are known to be the two main practical instances of nD-models [37]. Indeed, ILC is known to provide very interesting design strategies in various domains, the most famous one being robotics. Linear Matricial Inequalities (LMI) approaches to the stability analysis and stabilization of RP or ILC schemes have been developed. Besides, O. Bachelier and W. Paszke recently proposed the first "generic nD-version" of the celebrated Kalman-Yakubobvich-Popov (KYP) [32]. This powerful lemma extended to the nD case can be applied in multidimensional systems theory in order to tackle many problems encountered in the study of hybrid Roesser models. Soon after, an S-procedure approach to the study of Roesser models was developed, which is even more general than the KYP lemma [18]. Interestingly in this work, Roesser models are also subject to parametric uncertainties, under the form of implicit linear fractional representations, that makes it possible to develop robust controls. With such tools, based upon the resolution of LMI systems, one can propose a more systematic approach to the derivation of LMI conditions for numerous problems related to analysis and control of Roesser models. What remains to be understood is how to specialize the S-procedure to the particular case of RP and ILC. This important question will be investigated into during the mini-workshop.

3.4 Applications to spatially distributed systems

Finite-dimensional or lumped approximation of a spatially distributed system (e.g., a partial differential equation) generally produces a high dimensional system with a large number of inputs and outputs, which yields tedious control designs. The recent technological progress makes realizable the idea of microscopic devices with actuating, sensing, computing, and telecommunicating devices such as microelectromechanical systems (MEMS). The possibility to produce large arrays of such devices and to instrument systems governed by partial differential equations with them gives unprecedented capabilities for control (e.g., flow control for drag reduction [25], smart mechanical structures [5]). Moreover, networks of autonomous units with sensing and actuating capabilities (e.g., platoons [31, 14], automated highway systems [35, 39], airplane formation flight [42], satellite constellations [38]) have brought a renewal of interest in the study of distributed control design for spatially interconnected systems [4, 16, 17, 27], systems interconnected over a graph [26] or over a discrete symmetry group [36]. For different striking applications in this direction, see the videos on Raffaello D'Andrea's webpage http://raffaello.name/. The goal of this theme of research is to apply the different results and techniques developed in Sections 3.1 and 3.2 to systems defined by large numbers of interconnected systems or by spatially distributed systems (infinite, periodic, finite extent, boundary conditions) [27].

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5 Subvention

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