Constructive study of analysis and synthesis problems of multidimensional systems

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Using *computer algebra techniques* (e.g., Gröbner or Janet basis techniques [3, 17], cylindric algebraic decomposition [1, 7], computational real algebraic geometry [2]), the goal of the project is to develop a constructive study of *analysis and synthesis problems of multidimensional systems*. A multidimensional system (also called n-D systems) is a system in which information propagates in more than one independent direction (usually the time axis for standard 1-D systems) [5]. Multidimensional systems naturally arise in the study of partial difference equations, differential time-delay systems, partial differential equations, images, filters, ... [4, 5, 6, 9].

Within a frequency domain approach, a multidimensional system is defined by means of a *rational transfer matrix*, i.e., a matrix with entries in the field $\mathbb{R}(z_1, \ldots, z_n)$ of real rational functions in $z_1, \ldots, z_n$. The system is said to be *structurally stable* if the transfer matrix has no poles in the closed unit polydisc of $\mathbb{C}^n$, i.e., in:

$$\overline{\mathbb{D}}(z_1, 1) \times \ldots \times \overline{\mathbb{D}}(z_n, 1) = \{(z_1, \ldots, z_n) \in \mathbb{C}^n \mid |z_i| \leq 1, \ i = 1, \ldots, n\}.$$

The first goal of the project is to constructively study the ring of *structurally stable n-D systems*, i.e., the ring A of rational functions in $z_1, \ldots, z_n$ with no poles in the closed unit polydisc $\overline{\mathbb{D}}(z_1, 1) \times \ldots \times \overline{\mathbb{D}}(z_n, 1)$. This ring plays a central role in different problems studied in multidimensional systems theory [5, 6, 8, 9, 10, 11, 12, 13, 14, 22, 23] and time-delay systems [6, 9]. Algebraic properties of the ring A will be investigated. Important computational issues such as testing whether or not an element of $\mathbb{R}(z_1, \ldots, z_n)$ belongs to A [4, 8, 10], computing normal forms in the ring A/I, where I is a finitely generated ideal of A, or in a factor module, developing an effective Nullstellensatz, computing syzygy modules, ... will be investigated. A dedicated package will be developed in a computer algebra system (e.g., Maple, Mathematica).

A dictionary between properties of multidimensional systems (e.g., internal/strong/simultaneous stabilization, existence of (weakly) coprime factorizations) and properties of certain

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finitely generated modules or lattices over A has been developed [14, 15, 19, 20]. The second goal of the project is to develop a constructive study of the module structure of the ring A of structurally stable n-D systems (e.g., effective tests of the existence of torsion elements, of torsion-freeness, projectivity, stably freeness, freeness, invariants, extension modules) [16, 18]. Moreover, a *constructive version of Deligne's theorem* asserting that finitely generated projective A-modules are free [6, 9] will be investigated and algorithms for the computation of bases of finitely generated free A-modules will be developed. The computation of bases of free A-modules plays a fundamental role in the computation of Youla-Kučera parametrization [21] of all the stabilizing controllers of a structurally stabilizable system [14, 15]. The different results will be implemented in a dedicated computer algebra package.

Finally, the above techniques and results will be applied to analysis and synthesis problems of multidimensional systems and will be illustrated with important examples. In particular, the *strong and simultaneous stabilization problems* will be constructively studied [19, 22, 23].

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