

Modeling and Querying Evidential Databases: First Steps towards a Strong Representation System

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May 18th, 2017

Outline

- 1 Context and Background Materials
 - Typology of Imperfection
 - Theory of Evidence and Evidential Databases
- 2 Modeling Evidential Databases
 - Evidential Databases as Possible Worlds
 - Implementation of the Evidential Database Model
- 3 Querying Evidential Databases
 - Extended Relational Operators
 - Evidential Top- k Queries
- 4 Conclusion and Future works

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Imperfect Information

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- Uncertainty
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- John has **at least two** children and I am **not sure** about it
⇒ **Imprecise** and **Uncertain**

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⇒ **Precise** and **Uncertain**

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Theories and database models of Imperfection



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- Probability Theory
[Laplace, 1812]

Theories and database models of Imperfection



- Probability Theory
[Laplace, 1812]
- Possibility Theory
[Zadeh, 1978]

- Fuzzy sets Theory
[Zadeh, 1965]
- Evidence Theory
[Dempster, 1967; Shafer, 1976]



Probabilistic Databases

<i>ID</i>	<i>Weather</i>	<i>Probability</i>
1	rainy	0.6
2	sunny	0.4

(R.Cavallo, M.Pittarelli,1987)



<i>ID</i>	<i>Weather</i>	<i>Possibility</i>
1	rainy	1
2	sunny	0.4

Possibilistic Databases

[H.Prade, C.testemale, 1984]

**Evidential Databases**

<i>ID</i>	<i>Weather</i>	<i>Confidence Level</i>
1	Rainy 0.5 Sunny 0.5	[0.3 1]
2	{Sunny ; Rainy} 1	[0.5 0.5]

[S.K.Lee, 1992]

Theory of Evidence

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- Introduced by Dempster in 1967 and formalized by Shafer in 1976.
- Popularized by Smets in 1988 with [the Transferable Belief Model \(TBM\)](#).
- Theory of evidence provides an explicit representation of *uncertainty*, *imprecision* and *inconsistency*.

Evidential Databases

Evidential Databases

An *Evidential Database* (EDB) on its *compact form* has N tuples and D attributes. The value of an attribute c for an object I is called *evidential value*, V_{Ic} .

ID	Disease	Symptom	CL
1	Diabetes 1	Fatigue 0.4 {Fatigue, Nausea} 0.6	[0.3 ; 0.9]

Evidential Databases

Mass Function

A mass function, m , is a mapping from 2^Θ to $[0;1]$. The basic belief mass of an hypothesis x , $m(x)$ represents the degree of truth about that hypothesis x such that:

$$\sum_{x \subseteq \Theta} m^\Theta(x) = 1$$

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Evidential Databases

Confidence Level

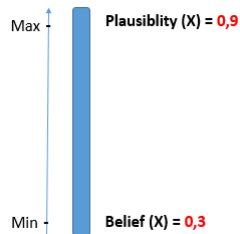
A confidence level CL is a measure that quantifies the degree of belief bel and plausibility pl about the existence of each tuple in the database,
 $CL = \{[bel, pl] | bel, pl \in [0, 1]; bel \leq pl\}.$

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Belief Function

The *belief function*, *bel*, is the minimal degree of belief given to an hypothesis x .

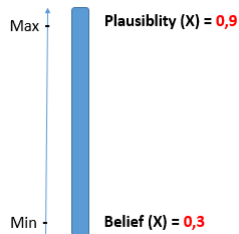
$$bel(x) = \sum_{y, x \subseteq \Theta: y \subseteq x} m(y)$$



Plausibility Function

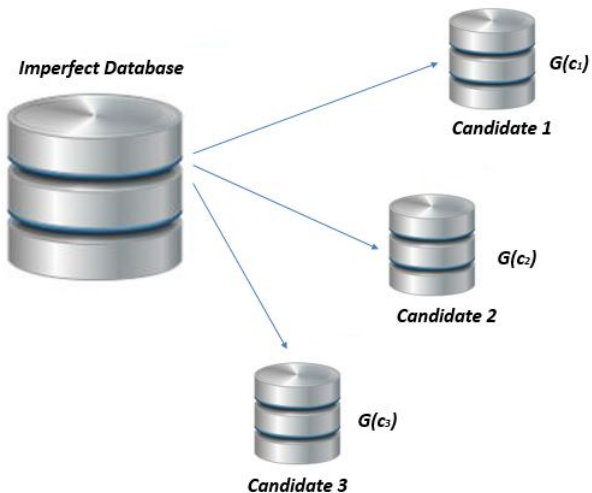
The *plausibility function*, pl , is the maximal amount of belief on the hypothesis x .

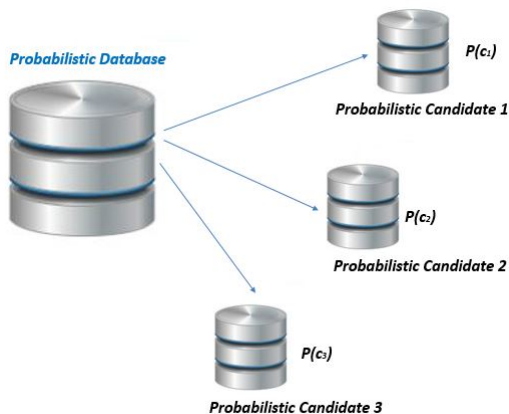
$$pl(x) = \sum_{y, x \subseteq \Theta: x \cap y \neq \emptyset} m(y)$$



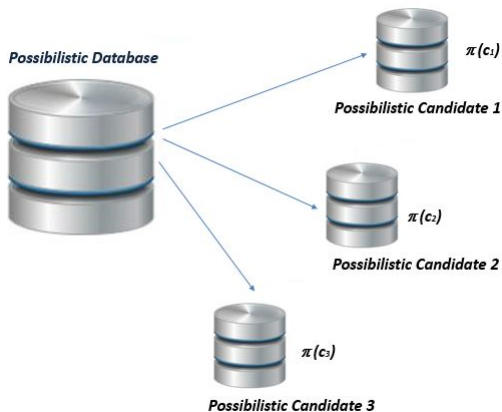
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[Abiteboul, Kanellakis and Grahne, 1991]



[Bosc, Liétard and Pivert, 2002]

Evidential Database



Generating Possible Worlds from an EDB: Example

EDB

ID	Disease	BloodType
1	Anemia 1	B 0.3 {B, O} 0.7
2	{Asthma, Flu} 1	A 1

Generating Possible Worlds from an EDB: Example

W_1	ID	Disease	BloodType
	1	Anemia	B
W_2	2	Asthma	A
	ID	Disease	BloodType
W_2	1	Anemia	B
	2	Flu	A

W_3	ID	Disease	BloodType
	1	Anemia	O
W_4	2	Asthma	A
	ID	Disease	BloodType
W_4	1	Anemia	O
	2	Flu	A

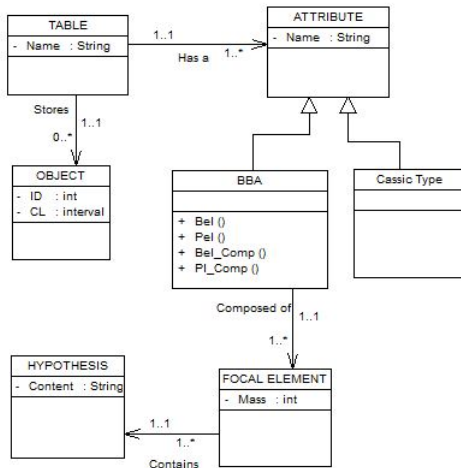
$$m(\{W_1; W_2\}) = 0.3 \quad m(\{W_1; W_2; W_3; W_4\}) = 0.7$$



Evidential Database

The non existence of an
implementation of an
Evidential Database in the
literature

Evidential Database Design



Implementation

SQL3 implementation:

- Facilitate the complex structure of an evidential database
- Optimize the I/O cost
- Accelerate the information extraction thanks to the indexes

Implementation

Table: Contribution of caches to queries re-execution

Database size	First execution time (s)	Next executions' times (s)
1000	0.2	0.03
5000	0.8	0.04
10 000	4.1	0.06
50 000	5.8	0.12
70 000	6.4	0.16
100 000	17	1.2

Outline

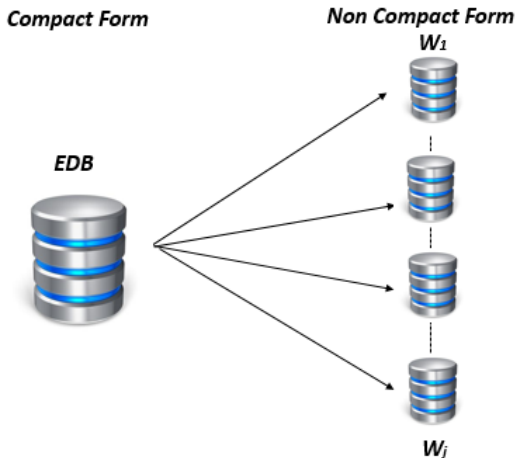
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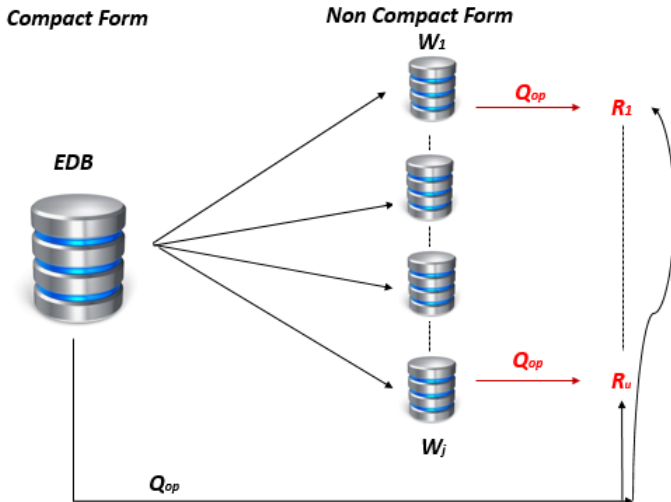


⇒ The compact form is the only feasible model in practice but the non-compact form is **fundamental** to prove if the model is a **Strong Representation System**.

Strong Representation System

A model is a *strong representation system* (SRS), when the result of querying *the compact form* is *equivalent* to the result of querying the set of *its possible worlds* (the non-compact form).





Querying Possible Worlds

Definition

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Confidence Level

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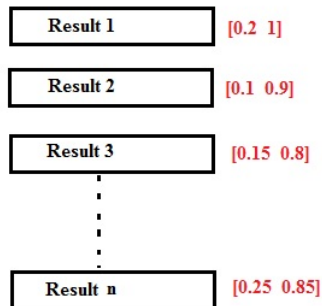
- Extended Evidential Select Operator
- Extended Evidential Project Operator

**Evidential Database**

Result 1	[0.2 1]
Result 2	[0.1 0.9]
Result 3	[0.15 0.8]
⋮	
Result n	[0.25 0.85]

Which are the k best results ?

How to rank them ?



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- Order queries' results in order to give only the most interesting answers.
- Top- k queries order answers using a score function that returns the k important answers with the highest scores.













H-index



H-index



H-index



ID	BookRate	CL
1	b_1 0.3 $\{b_2, b_3\}$ 0.7	[0.5 ; 1]
2	b_2 0.5 b_4 0.5	[0.3 ; 0.8]
3	$\{b_1, b_2, b_3\}$ 1	[1 ; 1]
4	b_3 1	[0.5; 0.9]

Top- k Query

The top-2 most appreciated books for readers ?

■ Introduced Evidential Score

item	EvidentialScore
b_1	$R_1 = [0.0375 ; 0.325]$
b_2	$R_2 = [0.0375 ; 0.525]$
b_3	$R_3 = [0.125 ; 0.65]$
b_4	$R_4 = [0.0375 ; 0.1]$

■ Adopted Preference Degree Equation

item	EvidentialScore
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Final Ranking

$b_3 \succ b_2 \succ b_1 \succ b_4$.

The Top-2 appreciated books are :

- b_3 with a confidence level $[0.125 ; 0.65]$
- b_2 with a confidence level $[0.0375 ; 0.525]$

- Defined A new Semantic for Imperfect Databases

Etop-2 Semantics

Books b_3 and b_2 are the 2 most appreciated **credible answers from the set of results**.

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Conclusion

- We introduced **the possible worlds' model** of an Evidential Database.

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- We implemented the Evidential Database model on its compact form.

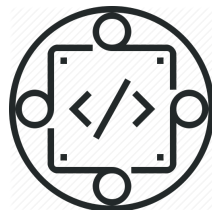
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- We introduced two extended relational evidential queries: [Selection and Projection](#).
- We introduced a new imperfect top- k query, called [Evidential Top- \$k\$ Query](#).

Future Works



- Implementation of Etop- k queries.

Future Works

- Proof of the Strong Representation System for Evidential Databases.

A	B
a_1 1	$\{b_1, b_2\}$ 0.2 b_1 0.8
$\{a_1, a_2\}$ 1	b_2 1



A	B	A	B
a_1	b_1	a_1	b_1
a_1	b_2	a_2	b_2
A	B	A	B
a_1	b_2	a_1	b_2
a_1	b_2	a_2	b_2

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Thank You