# Matlab routines: lsdtrpm, ivdtrpm

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17 novembre 2010

Version 2

The Matlab routine lsdtrpm computes the LS-estimates of discrete-time (DT) multi input multi output (MIMO) reinitialized partial moment (RPM) models. The Matlab routine ivdtrpm computes the LS-estimates with instrumental variable (IV) of DT MIMO RPM models. This report defines the DT RPM model and describes the implementation.

## 1 Discrete-Time RPM model

Consider an $n_a$-th order system defined by the transfer function

$$G(z) = \frac{b_0 + b_1 z^{-1} + \ldots + b_{n_b} z^{-n_b}}{1 + a_1 z^{-1} + \ldots + a_{n_a} z^{-n_a}}, \quad n_a \geq n_b \tag{1}$$

The true response of this system can be modeled, from the input-output measurements $\{u(k), y(k)\}$ with $k = 0, \ldots, N$, by the DT RPM model defined by

$$\widehat{y}(k) = \sum_{m=0}^{n_b} \widehat{b}_m \beta_m^u(k) + \sum_{n=1}^{n_a} \widehat{a}_n \alpha_n^y(k) + \gamma^y(k) \tag{2}$$

where

$$\begin{aligned}
&\beta_0^u(k) = \sum_{i=0}^{\widehat{K}-n_a} m_i u(k-i) \\
&\beta_m^u(k) = \beta_{m-1}^u(k-1),\ m = 1, \ldots, n_b \\
&\gamma^y(k) = -\sum_{i=1}^{\widehat{K}-n_a} m_i y(k-i) \\
&\alpha_1^y(k) = \gamma^y(k-1) - y(k-1) \\
&\alpha_n^y(k) = \alpha_{n-1}^y(k-1),\ n = 2, \ldots, n_a \\
&m_i = \frac{(i+1)(i+2)\ldots(i+n_a-1)A_{\widehat{K}-i}^{n_a}}{(n_a-1)!A_{\widehat{K}}^{n_a}} \\
&A_j^n = \frac{j!}{(j-n)!}
\end{aligned} \tag{3}$$

$m_i$ is the DT RPM FIR filter coefficients and $\widehat{K}$ is the design parameter called reinitialization interval.

The extension to the MIMO case with $n_y$ outputs and $n_u$ inputs is straightforward by considering $n_y$ MISO (multi input single output) models.

## 2 Parameter estimation

The DT RPM model (2) can be rewritten in a linear regression form

$$\widehat{y}(k) = \phi^T(k)\widehat{\theta}^{RPM} + \gamma^y(k) \tag{4}$$

where

$$\begin{aligned} \widehat{\theta}^{RPM} &= \left[\widehat{a}_1 \cdots \widehat{a}_{n_a} \widehat{b}_0 \cdots \widehat{b}_{n_b}\right]^T \\ \phi(k) &= \left[\alpha_1^y(k) \cdots \alpha_{n_a}^y(k) \beta_0^u(k) \cdots \beta_{n_b}^u(k)\right]^T \end{aligned} \tag{5}$$

The LS-estimate of $\widehat{\theta}^{RPM}$ is given by

$$\widehat{\theta}^{RPM} = \left[\sum_{k=\widehat{K}-n_a}^{N} \phi(k)\phi^T(k)\right]^{-1} \sum_{k=\widehat{K}-n_a}^{N} \phi(k)(y(k) - \gamma^y(k)) \tag{6}$$

## 3 Instrumental variable implementation

The instrumental variable iterative scheme can be used to remove the bias.

Consider the instrument built from an auxiliary model as follows

$$\xi(k) = \sum_{m=0}^{n_b} \widehat{b}_m u(k-m) - \sum_{n=1}^{n_a} \widehat{a}_n \xi(k-n) \tag{7}$$

Hence, the IV regressor is built

$$\zeta(k) = \left[\alpha_1^\xi(k) \cdots \alpha_{n_a}^\xi(k) \beta_0^u(k) \cdots \beta_{n_b}^u(k)\right]^T \tag{8}$$

The IV-estimate is given by

$$\widehat{\theta}^{IV} = \left[\sum_{k=\widehat{K}-n_a}^{N} \zeta(k)\phi^T(k)\right]^{-1} \sum_{k=\widehat{K}-n_a}^{N} \zeta(k)(y(k) - \gamma^y(k)) \tag{9}$$

A few iterations of the IV-estimate must be performed to remove the bias.

## 4 Choice of the design parameter

The DT RPM model requires the selection of a design parameter, the reinitialization interval, $\widehat{K}$.

Experiments show that the quality of the RPM model is not very sensitive to this choice. The selection of $\widehat{K}$ is not more difficult than the selection of the cutoff frequency and the order of the recommended data filter of an ARX model.

The design parameter $\widehat{K}$ allows the adaptation of the RPM model to the nature of the noise :

- If the perturbation is a white output-error noise, *i.e.* the structure of the system belongs to the OE model set, an optimal reinitialization interval exists, namely $\widehat{K}_{wn}$, for which the variance of the error is minimal and the bias is highly reduced. Many experiments led to the following conclusion : the parameter $\widehat{K}$ should be selected such that the interval $[0, \widehat{K}t_s]$, with $t_s$, the sampling period, is equivalent to the double of the main time constant for an aperiodic system or the double of the (zero to 90%) rising time for an oscillating system.
- If the perturbation is a white equation-error noise, *i.e.* the structure of the system belongs to the ARX model set, the reinitialization interval must be equal to $n_a$. Consequently, the RPM model is equivalent to an ARX model and the estimation is unbiased.
- If the perturbation is a coloured noise, *i.e.* the structure of the system does not belong to the ARX model set or the OE model set, the optimal reinitialization interval is on the interval $]n_a, \widehat{K}_{wn}[$.

In practice, the value of $\widehat{K}$ is selected as follows : $\widehat{K}$ is increased and a standard test, such as the quadratic criterion or the autocorrelation of the residuals, is evaluated to find the best $\widehat{K}$.